Do Greedy Autonomous Systems Make for a Sensible Internet?

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The Internet is a federation of thousands of autonomous systems. Understanding and modeling the interaction of the autonomous systems is a key to improved prediction and control of the Internet. Revenues less expenses for each of the autonomous systems, be they nonprofit or for profit, is a key factor determining the value the Internet. We explore several possible definitions of network equilibrium based on capacity and prices, with an eye towards exploring the effect of market pressures on the fairness of allocation of network resources.

(Joint work with Ganesh Gopal)

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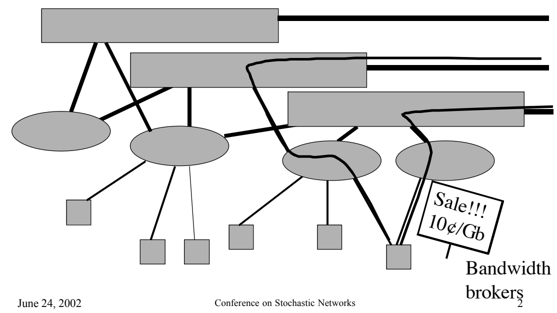
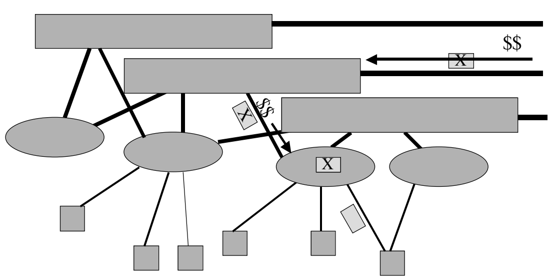
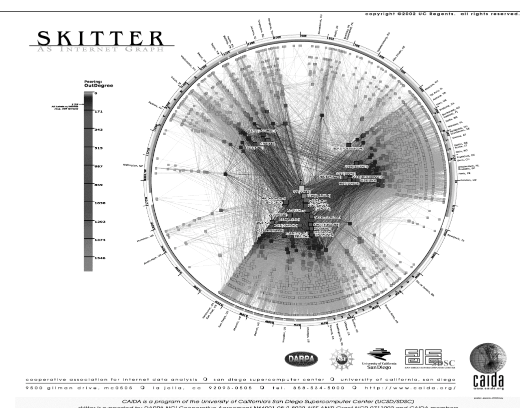


Paying Networks for Marks





General goal, vaguely stated

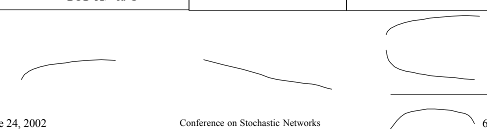
Handle general network topologies
Identify resource allocation guidelines
Investigate:
-Efficiency: Is maximum use made of the network?
-Fairness: How is network value distributed among players?

"Flash response" based on utility

Given a utility function U, the *flash response* to a price p is the value of rate x that maximizes U(x)-xp.

Let 0<□<∞

Utility function	Response function	Expenditure
U(x)=□(x^{1-□}-1)/(1-□)	x(p)=(□/p)^{1/□}	p□x(p)= □^{1/□} p^{1-(1/□)}
U(x)=□log(x) (if □=1)	x(p)= □/p	p□x(p)= □
U(x)=ax-(b/2)x^2 ** for x>a/b	x(p)=(a-p)_+ /b	x(p)=p(a-p)_+ /b

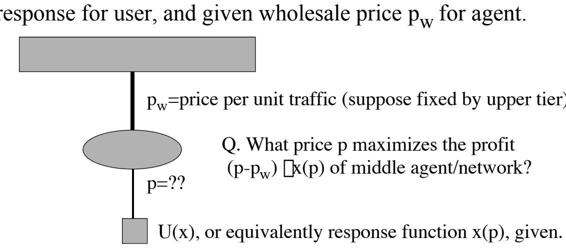


Outline of remainder of talk

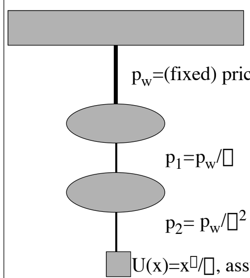
- Allocation based on "flash" responses to price
- Allocation based on "flash" responses to proportionally allocated capacity
- Allocation based on "considered" responses to proportionally allocated capacity

I. Allocation based on prices and flash responses

Can consider optimal price for middle agent, assuming flash response for user, and given wholesale price p_W for agent.



Approach can be extended to middle agents in series



p_w=(fixed) price per unit traffic

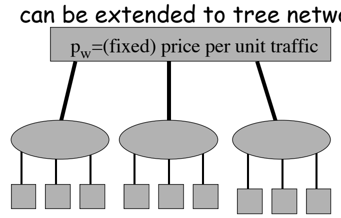
$p_1=p_w/\alpha$

$p_2=p_w/\alpha^2$

$U(x)=x^\alpha/\alpha$, assume $0<\alpha<1$

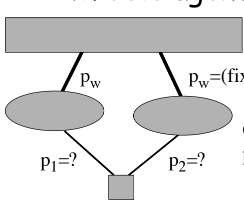
Each agent marks up the price by $1/\alpha$.
--Sum of profits is smaller than profit for a single middle agent.
--User pays higher price.
--Agent closer to user gets larger profit.

Allocation based on prices and flash responses can be extended to tree networks.



-End users may have a mixture of utility functions.
-Each network may have uniform pricing or targeted pricing, but overall effect is typically mixed unless per flow pricing is propagated through the network.
-Profit of middle networks might not be unimodal w/r price

Middle agents in parallel



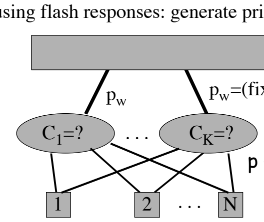
p_w=(fixed) price per unit traffic

Only Nash equilibrium gives zero profit to middle agents.
So what? Fixes?

U(x) given, use minimum price based routing

(i) If p_w's are even slightly different, then the network with the smaller p is used under Nash equilibrium.
(ii) Congestion -- links on a bottleneck are valuable--allocation of a finite resource kicks in. Price to fill.
(iii) Others: slow down oscillations through long-term contracts, delay in pricing information, etc.

II. Cournot viewpoint: a network announces the total Capacity, which is allocated in proportion to bids. (first try this using flash responses: generate prices so that supply meets demand.)



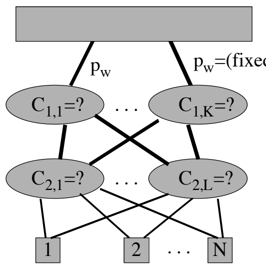
p_w=(fixed) price per unit traffic

p <-- determined so that supply=demand

End users have utility functions

Network i selects C_i to maximize $C_i(p-p_w)$.
Yields a K player game.

Cournot with flash users: more levels

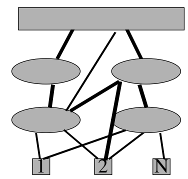


p_w=(fixed) price per unit traffic

Approach is to determine response functions from the bottom up. Result is typically a markup at each level.

$U(x)=x^{\square}/\square$, use price based routing

Middle agents in parallel (continued)

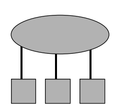


On "flash responses"

Advantages of the use of flash responses:
(1) simple to work with--in particular, given current price and current bid w_i, i^{th} user knows which direction to change bid.
(2) it suffices for the decomposition into network and user problems for the cooperative system optimization problem (Kelly, Malloo and Tan)
(3) many real users won't "game the system" anyway
(4) correct in limit of large numbers of users, in which effect of each user is small.

However, flash response approach may incorrectly predict incentives, especially for large network players.

III. Cournot allocation with "considered response"



Capacity C fixed.

User i bids w_i and receives payoff: $U(Cw_i/(w_1+\ldots w_n))- w_i$.

In giving a considered response, user takes into account the influence of its own bid on the price.

payoff = $U_i(Cw_i/(w_1+\ldots w_n))$- w_i

Nash equilibrium conditions for considered response (set w= $w_1+\ldots w_n$ and $x_i = Cw_i/w$):

$$U'\left(\frac{w_i C}{w}\right)\left[\frac{C}{w} - \frac{w_i C}{w^2}\right] = 1$$

which simplifies to

$$U'(x_i)\left[1 - \frac{x_i}{C}\right] = \frac{w}{C} \quad \text{or} \quad \tilde{U}'(x_i) = \frac{w}{C}$$

Factor due to "considered" response

price

User i payoff is $U(Cw_i/(w_1+\ldots w_n))$- w_i .

Or, the Nash equilibrium conditions are:

$$\tilde{U}_i'(x_i) = \frac{w}{C}$$

price

where the new utility function is defined by $\tilde{U}_i'(x) = U_i'(x)(1 - \frac{x}{C})$

--If U is increasing and concave, then the new function is strictly increasing and strictly concave on [0,C]. Gives existence and uniqueness of Nash equilibrium.

--New function involves C, so user does not know which way to adjust bid from knowledge of w_i and price alone.

Cash giveaway: an illustration of Cournot allocation with considered responses:

$10

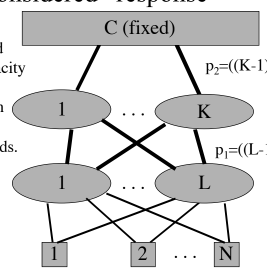
A B C

Players give money in A,B,C and receive a proportional share of the $10.

Nash equilibrium for cash giveaway assuming $U_i(x)=x$ for each player.

- -C dollars
- -Each player bids $C(n-1)/n^2$
- -Players each receive profit C/n^2
- -The bank retains C(n-1)/n dollars at the end of the day.

Sequential Cournot allocation with “considered” response



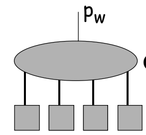
Each network and user bids for capacity from above. Allocation at each level is assumed proportional to bids.

L agents in a middle level each take profit=(revenue/L^2).

p_0=Cournot price, for end users bidding for shares of total bandwidth C on the basis of utility functions.

Given C, prices are determined bottom up.

Simple example with non-unimodal payoff functions.



Capacity C to be determined

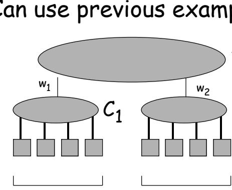
Users bid for capacity, which is proportionally allocated. Bids are “considered” responses. (Can also do for flash responses.)

$$U_1(x) = U_2(x) = 100(x - \frac{x^2}{2})$$

$$U_3(x) = U_4(x) = 2x - \frac{x^2}{200}$$

$$p_w = 1$$

Can use previous example to show multiple equilibria:



Two copies of the user set from previous example.

The two middle networks bid for capacity from top network, account for Cournot response of users below. This two person game has multiple Nash equilibria.

Recall our

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Problems

*The notions of equilibrium I've discussed involve the leader-follower philosophy. Not well suited to the mesh structure of the Internet.

*Most of the definitions yield multiple equilibria, even for the case of tree networks.

*Often profit functions to be maximized are not unimodal, defeating local stochastic approximation algorithms.