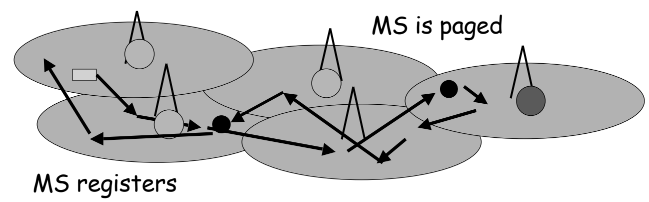
Bruce Hajek, "On Jointly Optimal Policies for Paging and Registration,"
Slides for talk at the *Workshop on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks* (WiOPT 2004), March 24-26 2004, Cambridge, UK. Available at www.uiuc.edu/~b-hajek
(Summarizes Hajek, Mitzel and Yang *INFOCOM 2003* and Hajek *Information Theory Workshop 2002*, and includes conjectures regarding 2-dimensional case.)

Abstract: This presentation explores optimization of paging and registration policies in cellular networks. Motion is modeled as a discrete-time Markov process, and minimization of the discounted, infinite-horizon average cost is addressed. The structure of jointly optimal paging and registration policies is investigated through the use of dynamic programming for partially observed processes. It is shown that there exist policies with a certain simple structure that are jointly optimal, though the dynamic programming approach does not directly provide an efficient method to find the policies.

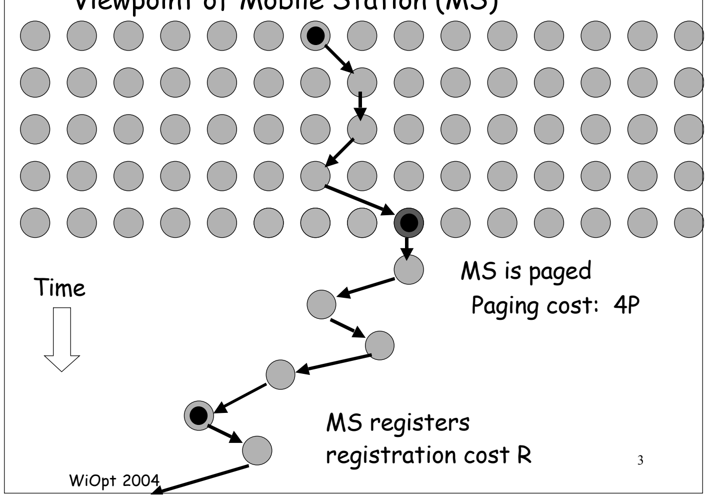
Jointly optimal paging and registration policies are identified for a cellular network composed of a linear array of cells. Motion is modeled as a random walk with a symmetric, unimodal step size distribution. Minimization of the discounted, infinite-horizon average cost is addressed. The jointly optimal pair of paging and registration policies is found. The optimal registration policy is a distance threshold type: the mobile station registers whenever its distance from the previous reporting point exceeds a threshold. The paging policy is ping-pong type: cells are searched in an order of increasing distance from the cell in which the previous report occurred. The existence of provably jointly optimal policies for other symmetric networks is addressed, and a connections to isoperimetric inequalities is made.

On Jointly Optimal Policies for Paging and Registration

Bruce Hajek
(based in part on joint work with Kevin Mitzel and Sichao Yang)



Viewpoint of Mobile Station (MS)



The given parameters and cost function

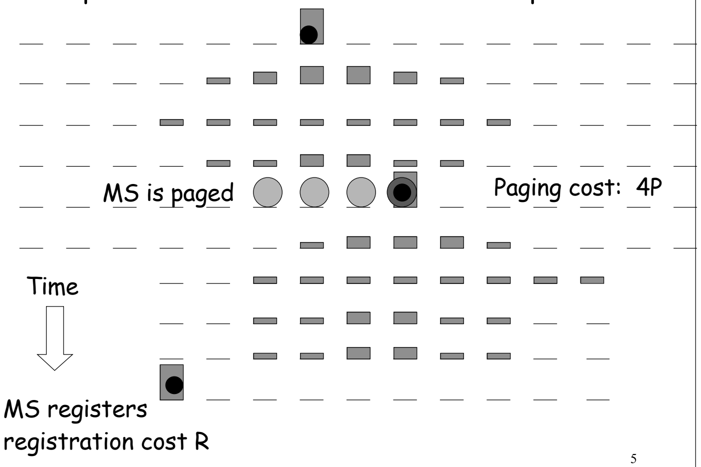
P= transition probability matrix for MS state
□$_p$-probability of being called in one slot
P - cost of paging one cell
R - cost of one registration
□- discount factor

Cost: C(u,v)=E[sum of □^{t}(cost at time t) over t=1,2, …]

paging policy   registration policy

The <u>joint optimization problem</u> is to find (u,v) such that C(u,v) is less than or equal to C(u′,v′) for all other pairs (u′,v′).

Viewpoint of the network -- conditional probabilities



Dynamic programming formulation

The joint optimization problem can be formulated as a dynamic programming problem on the space of conditional probability distributions (details in H., Mitzel, and Yang, Infocom 2003 paper).

An implication: there exist jointly optimal paging and registration policies specified by <u>reduced complexity laws</u>.

Reduced complexity laws (RCLs) f and g

Paging order=f(state at last report,
elapsed time since last report)

Registration probability=g(state at last report,
elapsed time since last report,
current state) in {0,1})

Write C(f,g) for cost when RCL pair (f,g) is used.

Finding an individually optimal pair of RCLs

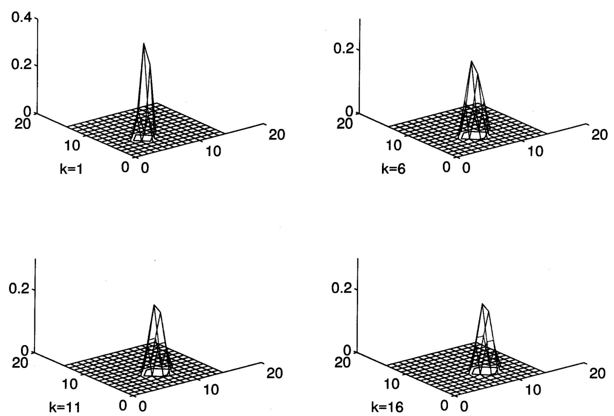
Observations about cost C(f,g):

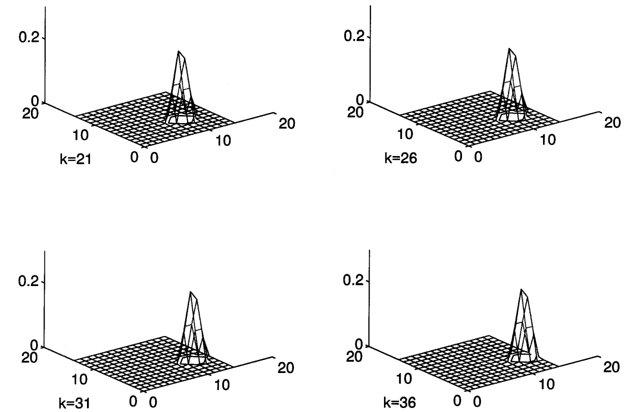
Given g, an optimal f is given by maximum likelihood search

Given f, an optimal g can be found by dynamic programming on a finite state space

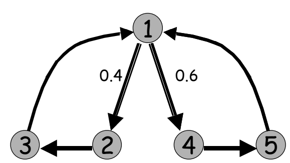
Suggests an iterative algorithm: $f^0 \rightarrow g^0 \rightarrow f^1 \rightarrow g^1 \rightarrow f^2 \ldots$

Provides good laws in numerical trials, though a simple example shows result may not be jointly optimal





Example of suboptimality



Search 4 then 2 if paged at time 1 or 4 or 7 … (optimal choice)

Suppose $0<R<\square\square_p P$ (so a registration costs less than risk of paging one cell one slot later.)

f - search 3 then 5 (at time 2 or 5 or 8 . . .)

g - register in state 4

(f,g) - is an individually optimal pair

f* - search 5 then 3 (at time 2 or 5 or 8 . . .)

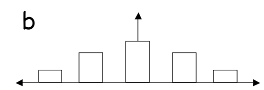
g* - register in state 2

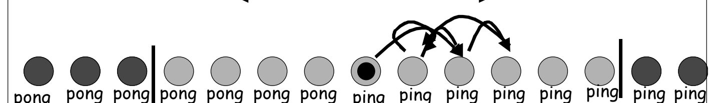
(f*,g*) - is a jointly optimal pair

Example of optimality

Suppose:

+ set of states is the set of integers, and that
+ MS takes independent steps with symmetric, unimodal distribution b





f* - ping-pong search from last sighting

g* - distance threshold registration policy, thresholds d and -d or -(d-1), for some positive integer d.

Proposition: (f*,g*) is optimal for some choice of thresholds Proof is outlined on the next five slides.

Example of optimality (continued)

Let □ and □ be summable vectors of nonnegative numbers.
Let $□_{[1]} \geqslant □_{[2]} \geqslant \cdots$ be the nonincreasing ordering of the □'s

Definition: □ <u>dominates</u> □, written $□ \succ □$ if:

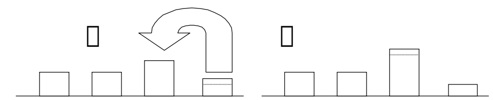
- (sum of the □'s) = (sum of the □'s)
- $□_{[1]} + □_{[2]} + \ldots + □_{[k]} \geqslant □_{[1]} + □_{[2]} + \ldots + □_{[k]}$ for all $k \geqslant 1$.

The vectors □ and □ are <u>equivalent</u>, and □ is said to be a <u>rearrangement</u> of □, if $□ \succ □$ and $□ \succ □$.

See Marshall and Olkin, *Inequalities: Theory of Majorization and Its Applications*, Academic Press, 1979.

Example of optimality (continued)

Intuitively, $□ \succ □$ if □ is more concentrated than □.
Transfer principle: If □ is obtained from □ by transferring mass from a smaller value of □ to a larger value, then $□ \succ □$:



Conversely, if $□ \succ □$, then □ can be obtained from □ by repeated transfers of mass.

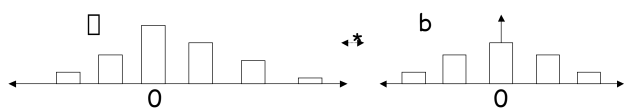
Definition: Let s(□)=mean number of pages needed to find an MS with probability vector □ for optimal search order:

$$s(□)=1+(1-□_{[1]}) + (1-□_{[1]}-□_{[2]}) + (1-□_{[1]}-□_{[2]}-□_{[3]}) + \cdots$$

Lemma: If $□ \succ □$ then $s(□) \leqslant s(□)$.

Example of optimality (continued)

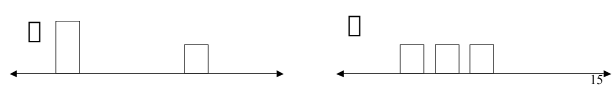
Definition: A probability vector □ on the integers is called <u>neat</u> if $□_0 \geqslant □_1 \geqslant □_{-1} \geqslant □_2 \geqslant □_{-2} \geqslant □_3 \geqslant □_{-3} \geqslant \cdots$ Equivalently, □ is neat if the ping-pong order is the optimal paging order for □.



Recall: b is the symmetric, unimodal step size distribution.

Lemma: If □ is neat, then □*b is neat.

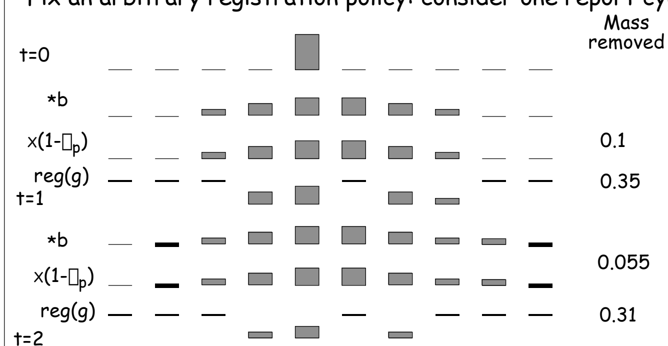
Lemma: If $□ \succ □$ and □ is neat, then $□*b \succ □*b$.



Example of optimality (continued)

Proof of proposition

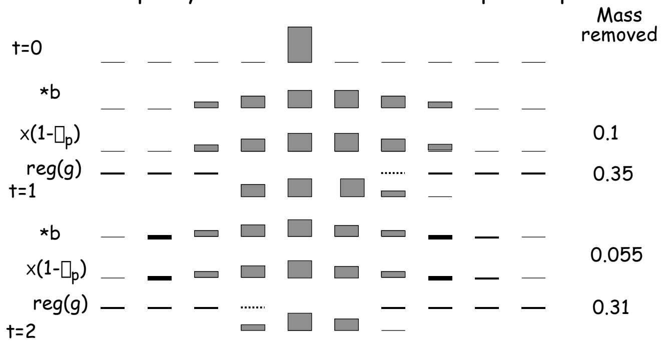
Fix an arbitrary registration policy: consider one report cycle



Idea: Compare to a threshold policy with same mass removed at each stage.

Proof of proposition

Threshold policy with same mass removed per step



By induction, distributions (w(k)) of this scheme are neat and dominate those of the original registration policy

Completion of proof of optimality:

The two strategies have same mass removed at each time, so they have the same mean registration cost.

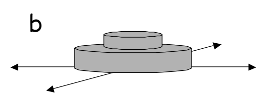
By induction, w(k) is neat for second strategy--> so ping-pong/threshold strategy has smaller mean paging cost.

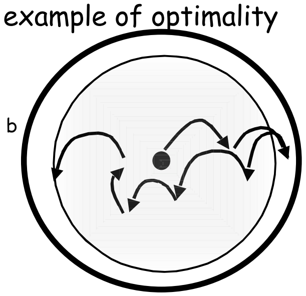
Thus, ping-pong paging can be used without loss of optimality.

The matching registration policy is fixed threshold type. □

Conjectured additional example of optimality

Suppose:
- set of states is plane
- MS takes independent steps with (radially) symmetric, unimodal density b





f* - expanding distance search from last sighting
g* - distance threshold registration policy

Conjecture: (f*,g*) is optimal for some choice of thresholds

Extension to 2 dimensions requires straight forward extension of concepts to continuous state space. For example:

Given functions f, g on the plane, nonnegative, integrable. say f dominates g ($f \succ g$) if for every c>0:

$$\sup_{A:\ \text{measure}(A)=c} \text{integral of } f \text{ over } A \;\geq\; \sup_{A:\ \text{measure}(A)=c} \text{integral of } g \text{ over } A$$

Say f is neat if f(x) is a nonincreasing function of |x| alone

Lemma: If f is neat, then f*b is neat.

Conjectured additional example (continued)

However, so far we have been unable to generalize one lemma, but each of the following conjectures would imply the one that follows it.

Conjecture A: If $f \succ \overline{f}$ and $g \succ \overline{g}$ and if f and g are neat, then $f*g \succ \overline{f}*\overline{g}$.

Conjecture B: Same as Conjecture A but with $g=\overline{g}$

Conjecture C: Expanding disk search and distance threshold registration policy are optimal if b is neat.

Note: Conjecture A (if true) implies the Brunn-Minkowski inequality for sets A,B in n dimensions: $m(A+B)^{1/n} > m(A)^{1/n} + m(B)^{1/n}$
where A+B={a+b: a in A and b in B}. (Take the sets to be the supports of the functions.)

Related work

C. Rose '96, '99
C. Rose & R. Yates '95
A. Bar-Noy & I. Kessler '93
A. Bar-Noy, I. Kessler, and M. Sidi, '96
U. Madhow, M.L. Honig, K. Stieglitz, '95

For citations and further reading

B. Hajek, IT Workshop, October '02
B. Hajek, K. Mitzel, and S. Yang, INFOCOM '03
www.uiuc.edu/~b-hajek

Conclusions on Paging and Registration

- Finding/proving joint optimality is hard, subtle
- Finding individually optimal pairs is fairly tractable
- Nevertheless, optimal polcies can always be expressed as ruduced complexity control laws
- Further analysis, such as exploration of approximate methods of dynamic programming may be useful.
- Models here are extremely simplistic, perhaps takehome message is that distance based policy may be reasonable. Systems today offer a quickly moving target for modelers.

Thank you.