On Variations of Queue Response for Inputs with the Same Mean and Autocorrelation Function

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***Abstract*—This paper explores the variations in mean queue length for stationary arrival processes with the same mean and autocorrelation functions, or equivalently, the same mean and power spectrum. Three types of processes, namely, two-state Markov-modulated Poisson processes, periodic-sequence modulated Poisson processes and processes generated by randomly filtering a white noise process, are investigated. Results show that the mean queue length can vary substantially for the first type of process, and can vary moderately for the second type of process, as the parameters of the processes are varied, subject to a specified mean and autocorrelation function. However, the mean queue lengths for the third type of arrival processes are determined by the input mean and autocorrelation functions. The results suggest that queueing performance can be hard to predict from spectral data alone when the power in low frequencies is large.**

***Index Terms*—Multiplexing delay, queueing, spectral analysis.**

I. Introduction

TRAFFIC STREAMS can be rather complex, so an important question is how to characterize them in a way that is easy to measure, and easy to relate to queueing performance. An approach, widely used through the years, has been to measure some statistics of a traffic stream, match the stream to a statistical model, and then base performance analysis and considerations, such as admission control and buffer sizing, on the selected model. For example, the interarrival distribution might be estimated, the arrival process modeled as a renewal process, and classical queueing theory used to derive bounds and approximations on queue lengths. Another characteristic that can be used is the mean and autocorrelation function of the arrival process; that is the focus of this paper. Since the processes we consider are assumed to be stationary, the autocorrelation function has the same information as the power spectrum.

Use of the spectrum of an arrival processes to help predict queue performance was extensively explored in [1]–[3]. The study [2] provides clear evidence that of four input statistics: 1) input power spectrum; 2) input bispectrum; 3) input trispectrum; and 4) one-dimensional marginal distribution, the input power spectrum is most essential to queueing analysis. Clear evidence is given in [1]–[3] that input power in the low frequency band has a dominant impact on queueing performance, whereas high-frequency power to a large extent can be neglected. Moreover, spectral characterization is an appealing tool for traffic modeling, because rich theories and techniques are available for the spectral estimation of stochastic processes, and spectral measurements are fairly easy to obtain. In addition, many traffic streams, such as video traffic with an underlying frame structure, exhibit nontrivial spectral structure.

The aim of this paper is to study in a clear and simple context the extent to which the mean and autocorrelation functions of an arrival process determine the mean length of a queue with a single, deterministic server (i.e., $\cdot/D/1$ type server). Given the promise of spectral methods for queuing analysis, practitioners would want to have rough guidelines about how well spectral data alone predicts queueing performance. They would also want to understand the extent to which spectral data specifies the statistics of a data sequence. Finally, they would want guidelines as to when spectral data is likely to need augmentation. Our findings regarding these issues are summarized in the final section.

The work [1], [2] focused mainly on the impact of input power spectral *variation*, especially in the low frequency region, on the queueing performance. In contrast, this paper mainly studies the impact of other input variations on the queueing performance for a *given* input power spectrum. Recent related work [5] indicates that use of the one-dimensional marginal distribution of the rate, can be used in conjunction with spectral statistics, to improve predictions in the case of processes with large power in the low frequency range.

Three different types of random processes are explored. Two-state Markov modulated Poisson (MMP) processes and periodic-sequence modulated Poisson (PSMP) processes are considered in Sections II and III. These types of processes were both considered in [1] and are both special cases of the discrete version of the circulant modulated Poisson processes investigated in [2]. A large class of randomly filtered white noise processes is considered in Section IV. Conclusions are given in Section V. In closing this section, we remark that the mean values of the random processes considered in this paper are determined by the power spectra (see the Appendix). Therefore, it is not necessary for us to explicitly match the means of the processes, but we do so for clarity.

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II. Queue Responses for Two-State MMP Processes

MMP processes, in which the modulating Markov process has two states, are the arrival processes investigated in this section. First such processes are described, and then the variation in queue response is explored.

A. Source Model

A discrete time MMP process $A = (A_k)$ can be described as follows. There is a stationary discrete time Markov process $\Theta = (\Theta_k)$, and for each possible state i of Θ there is a specified mean number of arrivals γ_i. The *rate process* $M = (M_k)$ for A is defined by $M_k = \gamma_{\Theta_k}$. Given M, the variables A_k are conditionally independent Poisson random variables with $E[A_k|M] = M_k$. In this section the Markov process Θ is taken to have states 0 and 1, with one-step transition probability matrix P written as

$$P = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}.$$

The associated input rate vector is denoted by $\vec{\gamma} = [\gamma_0, \gamma_1]$. In this two-state case, the rate process M itself is Markov.

The mean and autocorrelation functions of the rate process M are given by

$$\overline{\gamma} = \pi_0\gamma_0 + \pi_1\gamma_1 = \frac{\beta}{\alpha+\beta}\gamma_0 + \frac{\alpha}{\alpha+\beta}\gamma_1 \tag{2.1}$$

$$\begin{aligned} R_M(n) &= E[M_0 M_n] = \sum_{i=0}^{1}\sum_{j=0}^{1} \gamma_i\gamma_j P_{ij}^{|n|}\pi_i \\ &= \overline{\gamma}^2 + \alpha\beta\left(\frac{\gamma_1-\gamma_0}{\alpha+\beta}\right)^2 (1-\alpha-\beta)^{|n|} \end{aligned} \tag{2.2}$$

where $[\pi_0, \pi_1]$ is the steady-state probability distribution for P. Since $R_M(0) = \text{var}(M_0) + \overline{\gamma}^2$, R_M can be expressed as

$$R_M(n) = \overline{\gamma}^2 + \psi\lambda^{|n|} \tag{2.3}$$

where ψ is the variance of M_k and λ is the smaller eigenvalue of P, i.e., $\lambda = 1 - \alpha - \beta$.

It is easy to show that $E[A_k] = E[M_k]$, and the autocorrelation function of the arrival process A is related to that of M by $R_A(n) = R_M(n) + \overline{\gamma}\delta_n$where δ_n denotes

$$\delta_n = \begin{cases} 1, & \text{if } n = 0 \\ 0, & \text{else.} \end{cases} \tag{2.4}$$

The term δ_n accounts for the wide-sense white noise induced by the modulation operation. In the frequency domain, this additional term in R_A simply corresponds to adding a constant term $\overline{\gamma}$ to the power spectrum of M. Therefore, R_A and R_M are nearly equivalent. For convenience, we specify R_M instead of R_A in this section.

B. Matching Input Autocorrelation Functions

By (2.3), the mean and autocorrelation function of a two-state MMP process is completely characterized by three parameters: $\overline{\gamma}$, ψ, and λ. However, one needs four parameters, α, β, γ_0, and γ_1, to specify such a process. Therefore, for any given autocorrelation function in the general form of (2.3), one can synthesize infinitely many different arrival processes with the one extra degree of freedom.

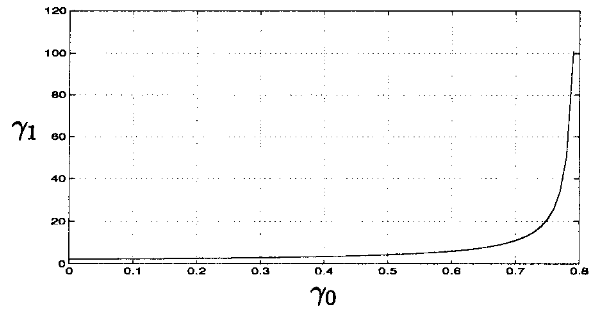

Fig. 1. γ_1 versus γ_0, given $\overline{\gamma} = 0.8$, $\psi = 1$, and $\lambda = 0.25$.

Use (2.1) and match (2.2) and (2.3) to obtain a set of three equations:

$$\begin{cases} \dfrac{\alpha\gamma_1 + \beta\gamma_0}{\alpha+\beta} = \overline{\gamma} \\ \alpha + \beta = 1 - \lambda \\ \alpha\beta\left(\dfrac{\gamma_0-\gamma_1}{\alpha+\beta}\right)^2 = \psi \end{cases} \tag{2.5}$$

with four constraints:

$$0 \le \alpha \le 1, \quad 0 \le \beta \le 1, \quad \gamma_0 \ge 0, \quad \gamma_1 \ge 0. \tag{2.6}$$

Choose γ_0 as the free variable and solve the above equations to yield

$$\begin{cases} \gamma_1 = \dfrac{\psi}{\overline{\gamma}-\gamma_0} + \overline{\gamma} \\ \alpha = \dfrac{(1-\lambda)(\overline{\gamma}-\gamma_0)}{\gamma_1-\gamma_0} \\ \beta = \dfrac{(1-\lambda)(\gamma_1-\overline{\gamma})}{\gamma_1-\gamma_0} \end{cases} \tag{2.7}$$

with constraints

$$\begin{cases} \gamma_0 < \overline{\gamma}, & \text{for } \lambda \ge 0 \\ \overline{\gamma} - \sqrt{\dfrac{\psi}{-\lambda}} \le \gamma_0 < \overline{\gamma}, & \text{for } \lambda \le 0 \end{cases} \tag{2.8}$$

assuming $\gamma_0 < \overline{\gamma} < \gamma_1$. According to these equations, γ_0 can take any value in $[0, \overline{\gamma})$. As γ_0 converges to $\overline{\gamma}$, γ_1 increases toward ∞, α decreases to 0 while β approaches $1-\lambda$. This shows how vastly different two arrival processes can be while having the same mean and autocorrelation functions. Figs. 1 and 2 show how γ_1, α, and β vary as γ_0 ranges over $[0, \overline{\gamma})$, given $\overline{\gamma} = 0.8$, $\psi = 1$, and $\lambda = 0.25$.

Note that if we restrict attention to two-state "on-off" sources by fixing $\gamma_o = 0$, then the model is completely determined by the autocorrelation function. In the general case one could decompose the arrival process as the sum of an on–off process with rates 0 and $\gamma_1 - \gamma_0$, and a Poisson process with fixed rate γ_0. The variation of the average queue length as investigated in the next section is solely due to variations in γ_0, the fixed rate portion of the total arrival process.

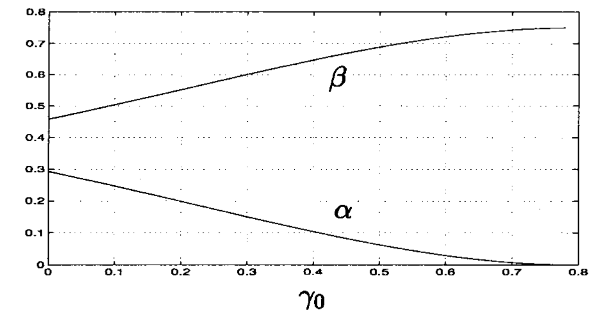

Fig. 2. α and β versus γ_0, given $\overline{\gamma} = 0.8$, $\psi = 1$, and $\lambda = 0.25$.

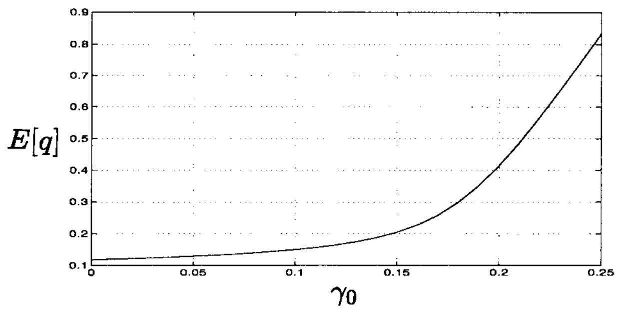

Fig. 3. Mean queue length versus γ_0, given $\overline{\gamma} = 0.25$, $\psi = 0.0625$, and $\lambda = 0.9$.

C. Queue Responses

Consider a discrete time $\cdot/D/1$ queue with unit service time, with the two-state MMP arrival process A described above. The mean queue length $E[q]$ is readily calculated, since the queue is an $M/G/1$ phase type process [6], [7]. The equations we used can be found in [8] and are omitted here. Rather, some numerical results will be described.

Fig. 3 shows the variation of mean queue length as a function of γ_0. As seen from the plot, γ_0 plays an important role in the queue response. When γ_0 is small (within the range of $[0, 0.1]$), the variation of the mean queue length $E[q]$ is also relatively small. However, as γ_0 increases toward $\overline{\gamma}$, $E[q]$ increases quickly to reach the maximum. To quantitatively describe the variation of $E[q]$, define

$$\delta(\overline{\gamma}, \psi, \lambda) \triangleq \frac{\max(E[q]) - \min(E[q])}{\min(E[q])}$$

, where the min and max are taken with respect to γ_0. The data for Fig. 3 yield $\delta(0.25, 0.0625, 0.9) = 6.11$. While elsewhere in this paper we focus on the mean queue length, we note for this example that the second moment and variance of the queue length exhibit a very large variation, since they both tend to infinity as $\gamma_0 \nearrow \overline{\gamma}$.

In order to examine the maximum possible variation of the mean queue length, we plot $\delta(\overline{\gamma}, \psi, \lambda)$ for fixed λ in Fig. 4, and for fixed ψ in Fig. 5. Fig. 4 indicates that δ is large in the region where both ψ and $\overline{\gamma}$ are small. However, in this region $E[q]$ is small, so that the large values of δ (in which differences are normalized by min $E[q]$) might not be significant in applications. To focus on the variation more fully, consider Fig. 6, which shows both the maximum and minimum values of $E[q]$ for λ fixed. For example, for $\overline{\gamma} = 0.4$ and $\psi = 0.2$, we find that $E[q]$ ranges from 0.81 to 3.2, as γ_o varies. This is a factor of four difference in mean queue length for the same mean and autocorrelation of the input, but we might term $E[q]$ as being moderate throughout this range. For a given ψ, Fig. 5 indicates that δ is large for moderate values of $\overline{\gamma}$ and λ close to unity. In this case, both the relative and absolute variation of $E[q]$ for a fixed mean and autocorrelation function of the input can be significant. To illustrate the variation of queue response under an extreme case, Fig. 7 shows the mean queue length with $\overline{\gamma} = 0.2$, $\psi = 0.1$, and $\lambda = 0.998$, which leads to $\delta = 268.39$.

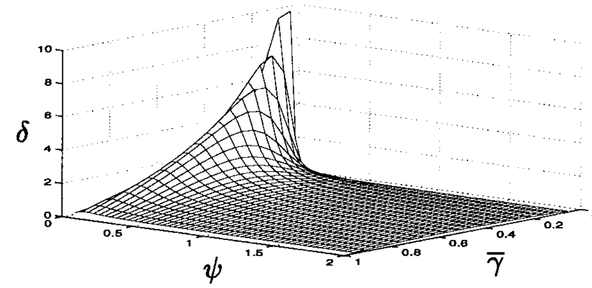

Fig. 4. $\delta(\overline{\gamma}, \psi, \lambda)$ as a function of $\overline{\gamma}$ and ψ, with $\lambda = 0.9$.

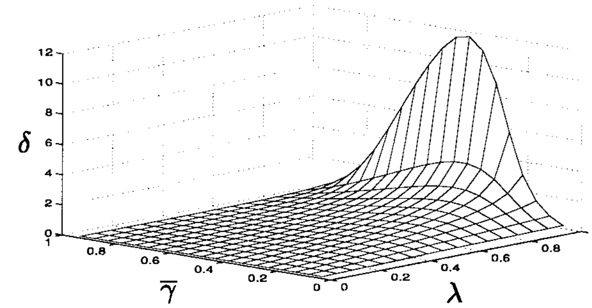

Fig. 5. $\delta(\overline{\gamma}, \psi, \lambda)$ as a function of $\overline{\gamma}$ and λ, with $\psi = 0.25$.

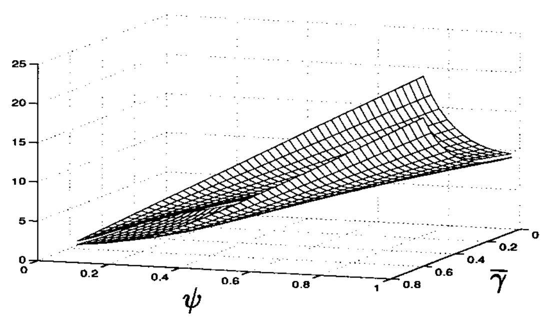

Fig. 6. min($E[q]$) and max($E[q]$), as a function of $\overline{\gamma}$ and ψ, for $\lambda = 0.9$.

The largest relative differences in mean queue lengths (i.e., largest values of δ) are observed for λ near one. This is the case that there is the most power in low frequencies, or equivalently, the longest range statistical correlations.

Note in Figs. 4 and 5 that for heavy loads (i.e., $\overline{\gamma}$ near one), the variation is not large. An explanation for this is that in heavy traffic the arrival process is averaged over long time periods by the queue. Therefore, the central limit theorem (i.e., diffusion approximation) holds, so that the response is mainly determined by the mean and autocorrelation function of the arrival process.

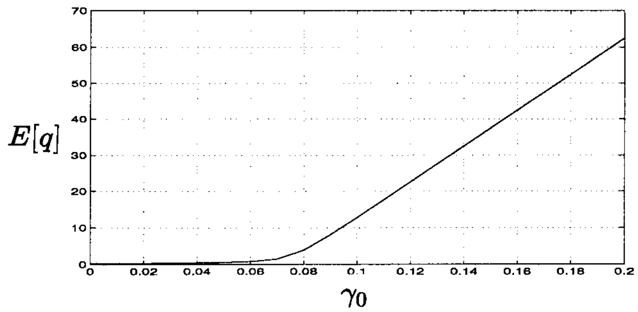

Fig. 7. Mean queue length versus γ_0 in an extreme case: $\overline{\gamma} = 0.2$, $\psi = 0.1$, and $\lambda = 0.998$.

III. Queue Responses for PSMP Processes

In this section, we first describe the model for a PSMP process and give its mean, autocorrelation function and power spectrum. Then we show how to synthesize different PSMP processes with the same mean and autocorrelation functions through phase manipulation, and examine the resulting variation of the queue responses.

The paper [1] examined a different approach for generating PSMP processes with the same mean and autocorrelation function. The rate process was taken to be the sum of two sinusoidal inputs, and variations of the phases of the two inputs were explored. It was found that the effect of changing the phases is negligible unless the frequencies of the two signals are harmonically related, in which case the mean queue length varied by up to 25% as the phases varied. Larger variations are found for the phase perturbation method explored in this section.

A. Source Model

A discrete-time PSMP process $A = (A_k)$ is a doubly stochastic process modulated by a rate process $M = (M_k)$. Given M, the variables A_k are conditionally independent Poisson random variables with $E[A_k|M] = M_k$. The process M itself is a random periodic sequence with period N, defined as

$$M_k = \gamma_{k+\Omega \bmod N}$$

for some finite sequence $\vec{\gamma} = [\gamma_0, \gamma_1, \cdots, \gamma_{N-1}]$ and a random variable Ω uniformly distributed on $\{0, 1, \cdots, N-1\}$.

A PSMP process is also a MMP process as described in Section II, since the rate process M can be written as $M = \gamma_{\Theta_k}$, where Θ_k is a Markov process with the ring-type deterministic transitions illustrated in Fig. 8.

The mean and autocorrelation functions of the rate process M are given by

$$E[M_k] = E[\gamma_{k+\Omega \bmod N}] = \frac{1}{N}\sum_{k=0}^{N-1}\gamma_k \triangleq \overline{\gamma} \tag{3.1}$$

$$R_M(n) = E[\gamma_\Omega \gamma_{n+\Omega \bmod N}] = \frac{1}{N}\sum_{i=0}^{N-1}\gamma_i\gamma_{(i+n) \bmod N}. \tag{3.2}$$

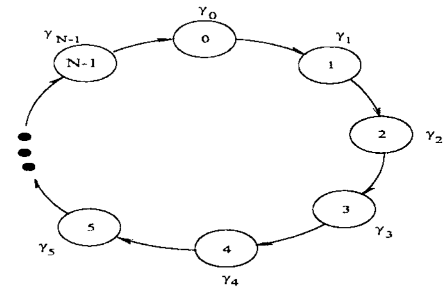

Fig. 8. State transition diagram of a ring-type periodic Markov chain.

Therefore, the mean of M equals the arithmetic average of the finite sequence $\vec{\gamma}$, and the autocorrelation of M equals the periodic autocorrelation function of $\vec{\gamma}$.

The power spectrum of M, which is defined as $1/N$ times the discrete Fourier transform (DFT) of one period of R_M, can be found as

$$\begin{aligned} P_k &= \frac{1}{N}\sum_{n=0}^{N-1} R_M(n) e^{-j(2\pi k n/N)} \\ &= \frac{1}{N^2}\sum_{n=0}^{N-1}\sum_{i=0}^{N-1}\gamma_i\gamma_{(i+n) \bmod N} e^{-j(2\pi k n/N)} \\ &= |\hat{\gamma}_k/N|^2 \end{aligned} \tag{3.3}$$

where $\hat{\gamma}$ is the DFT of $\vec{\gamma}$. The power spectrum is normalized so that the sum of the P_k is $R_M(0)$, the total power.

As noted in the previous section, the mean of the arrival process A equals $E[M_k] = \overline{\gamma}$, and its autocorrelation function R_A is given by $R_A(n) = R_M(n) + \overline{\gamma}\delta_n$. For convenience, as in the previous section, we focus on R_M instead of R_A in this section.

B. Matching Input Power Spectra

By (3.3), the power spectrum of the rate process M is completely specified by the magnitude of $\hat{\gamma}$. Therefore, we can synthesize a given input power spectrum by two different rate processes, constructed from two finite sequences which have DFT's with the same magnitude but different phase spectra.

Consider a special case of $\vec{\gamma}$ defined as

$$\vec{\gamma} = [a, \underbrace{b, b, \cdots, b}_{N-1}].$$

Its DFT is

$$\begin{aligned} \hat{\gamma}_k &= \sum_{i=0}^{N-1}\gamma_i e^{-j(2\pi k i/N)} \\ &= \begin{cases} a + (N-1)b = N\overline{\gamma}, & k = 0 \\ a - b, & \text{else.} \end{cases} \end{aligned}$$

Assuming $N \geq 3$, we change the phases of $\hat{\gamma}_m$ and $\hat{\gamma}_{M-m}$ by an amount θ for a single index m with $1 \leq m \leq \lfloor (N-1)/2 \rfloor$. The resulting perturbation of $\hat{\gamma}$, denoted by $\hat{\gamma}^{m,\theta}$, is

$$\hat{\gamma}_k^{m,\theta} = \begin{cases} N\overline{\gamma}, & k = 0 \\ (a-b)e^{j\theta}, & k = m \\ (a-b)e^{-j\theta}, & k = N-m \\ a-b, & \text{else.} \end{cases}$$

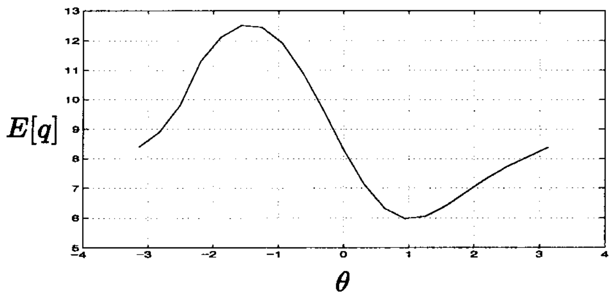

Fig. 9. Mean queue length as a function of θ with $N = 150$, $\overline{\gamma} = 0.85$, $m = 1$, and $b = 0.8\overline{\gamma}$.

Taking the inverse DFT of $\hat{\gamma}^{m,\theta}$, we have a new finite sequence denoted by $\vec{\gamma}^{m,\theta} = [\gamma_0^{m,\theta}, \cdots, \gamma_{n-1}^{m,\theta}]$

$$\begin{aligned}\gamma_i^{m,\theta} &= \frac{1}{N}\sum_{k=0}^{N-1}\hat{\gamma}_k^{m,\theta}e^{j(2\pi ki/N)}\\ &= \gamma_i - \frac{4(a-b)}{N}\sin\left(\frac{2\pi mi}{N}+\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\end{aligned} \tag{3.4}$$

for $i = 0, 1, \cdots, N-1$. For $\vec{\gamma}^{m,\theta}$ to be a valid arrival rate sequence, $\gamma_i^{m,\theta}$ has to be nonnegative for $i = 0, 1, \cdots, N-1$. A sufficient condition on a and b, allowing us to vary m and θ over the full range $1 \leq m \leq \lfloor (N-1)/2 \rfloor$ and $-\pi \leq \theta \leq \pi$, is that $a - |4(a-b)/N| \geq 0$ (implying $\gamma_0^{m,\theta} \geq 0$) and $b - |4(a-b)/N| \geq 0$ (implying $\gamma_i^{m,\theta} \geq 0$ for $i \neq 0$). Equivalently, it suffices that

$$\frac{4a}{N+4} \leq b \leq \frac{(N+4)a}{4}$$

or in terms of $\overline{\gamma}$ and b,

$$\frac{4}{5}\overline{\gamma} \leq b \leq \frac{N(N+4)}{N(N+4)-4}\overline{\gamma}. \tag{3.5}$$

In summary, if $\overline{\gamma}$, b, and N satisfy condition (3.5), then for any m and θ as above, $\vec{\gamma}^{m,\theta}$ is a nonnegative sequence with $\hat{\gamma}_0 = \hat{\gamma}_0^{m,\theta}$ and $|\hat{\gamma}| = |\hat{\gamma}^{m,\theta}|$. The synthesized PSMP process thus has the same mean and power spectrum for all m and θ.

C. Queue Responses

Consider a discrete time $\cdot/D/1$ queue with unit service time, with a PSMP arrival process as described. We again used the theory of phase-type $M/G/1$ processes [6]–[8] to evaluate the mean queue length $E[q]$ with the mean and autocorrelation functions of the arrival process fixed.

Fig. 9 shows the mean queue length as a function of θ with $N = 150$, $m = 1$, $\overline{\gamma} = 0.85$, and $b = 0.8\overline{\gamma}$. To quantitatively describe the variation of $E[q]$, define

$$\delta(N, m, \overline{\gamma}, b) \triangleq \frac{\max_\theta(E[q]) - \min_\theta(E[q])}{\min_\theta(E[q])}.$$

The data for Fig. 9 yield $\delta(150, 1, 0.85, 0.68) = 1.093$. Fig. 10 shows δ as a function of m with $N = 150$, $\overline{\gamma} = 0.9$, and $b = 0.8\overline{\gamma}$, in which δ drops sharply as m increases. This is because as m increases, we are modifying the phase of a higher frequency component of $\vec{\gamma}$, so the effect on $E[q]$ becomes less important. This is why, although we could vary m and θ arbitrarily, we restrict attention in what follows to the case $m = 1$. We examine how each of the three parameters, N, $\overline{\gamma}$, and b, may affect δ. Note that the input power spectrum also changes as N, γ or b varies.

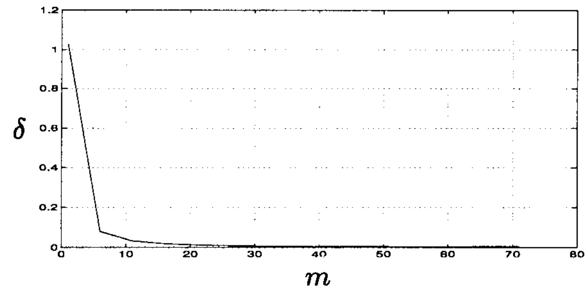

Fig. 10. δ as a function of m, with $N = 150$, $\overline{\gamma} = 0.9$, and $b = 0.8\overline{\gamma}$.

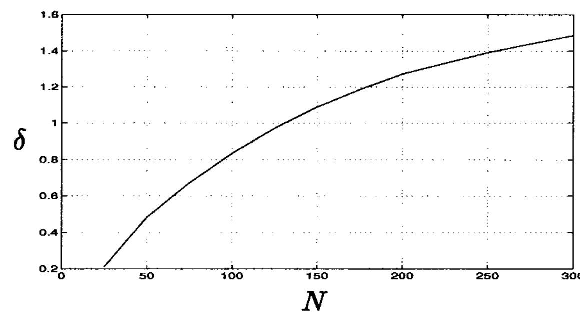

Fig. 11. δ as a function of N, with $\overline{\gamma} = 0.9$, $m = 1$, and $b = 0.8\overline{\gamma}$.

Fig. 11 shows δ as a function of N with $\overline{\gamma} = 0.9$, $m = 1$, and $b = 0.8\overline{\gamma}$. A fluid-limit analysis can be used to show that δ converges as $N \to \infty$, and the limit is roughly 2.5, representing a 3.5-fold difference between minimum and maximum mean queue sizes. (The rate of convergence is slow.) Note that the DC power in the arrival process is $P_0 = \overline{\gamma}^2$, whereas for large N the power at other frequencies is given by $P_k \approx 0.04\overline{\gamma}^2$ for $1 \leq k \leq N-1$. Thus, the ratio of power at the DC level to total power tends to zero as N tends to infinity. Basically, for large N, there is a large (order of N) burst of arrivals once every N time slots, causing a nearly flat power distribution. The variations of θ do not effect the power spectrum, but effect the phase of the lowest nonzero frequency component of the process M.

Fig. 12 shows δ as a function of $\overline{\gamma}$ with $N = 150$, $m = 1$, and $b = 0.8\overline{\gamma}$. For $\overline{\gamma} \leq 0.85$, δ increases as $\overline{\gamma}$ increases, as expected. However, δ decreases for larger values of $\overline{\gamma}$. An explanation for this is that the diffusion approximation becomes valid under heavy load conditions (i.e., $\overline{\gamma}$ close to 1), so that the queue response is mainly determined by the input mean and autocorrelation functions.

Fig. 13 shows δ as a function of b with $N = 150$, $\overline{\gamma} = 0.9$, and $m = 1$. By (3.5), the allowable range of b is $[0.8\overline{\gamma}, 1.0002\overline{\gamma}]$. It shows that δ drops from its maximum, which is achieved at $b = 0.8\overline{\gamma}$, to zero as b approaches $\overline{\gamma}$. This is because if $b = \overline{\gamma}$, then $\vec{\gamma}_k = 0$ unless $k = 0$, so that no phase change is possible.

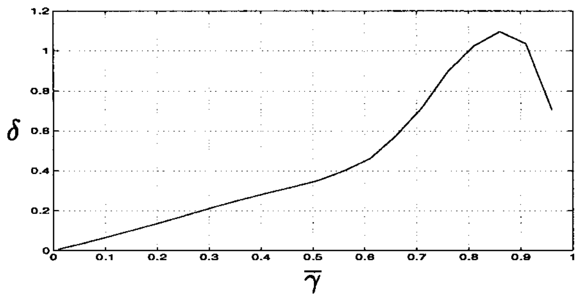

Fig. 12. δ as a function of $\overline{\gamma}$ with $N = 150$, $m = 1$, and $b = 0.8\overline{\gamma}$.

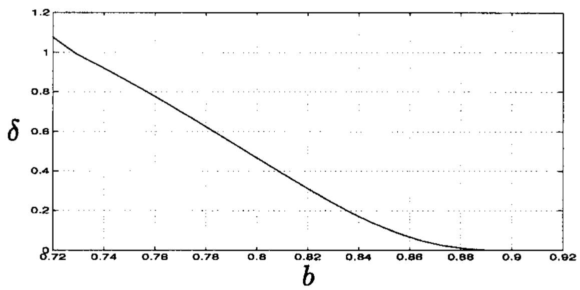

Fig. 13. δ as a function of b with $N = 150$, $\overline{\gamma} = 0.9$, and $m = 1$.

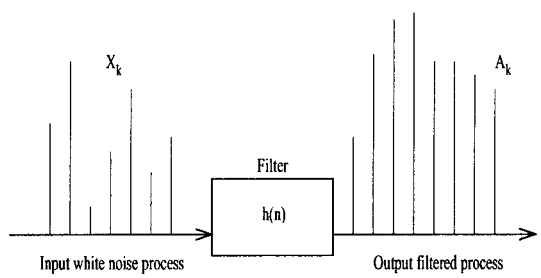

Fig. 14. Model for the generation of a randomly filtered arrival process.

IV. Queue Responses for Processes Generated by Random Filtering

In this section, arrival processes generated by random filtering are described and corresponding expressions for the mean length of an $\cdot/D/1$ are given. Finally, the relationship between input autocorrelation function and the mean queue length is examined for a subclass of the processes.

A. Source Model

A randomly filtered process $A = (A_k)$ can be generated by passing a white noise process $X = (X_k)$ through a filter with random coefficients. The coordinates of both X and the filter take values in Z_+. Fig. 14 shows the basic model for the generation of a randomly filtered process. For the purposes of exposition, we think of there being X_k *messages* initiated at time k, such that each message stimulates a set of *packet* arrivals, according to the random filter.

The filter is denoted by $(h_{k,u,j}\colon k,\, u \in Z,\, j \in Z_+)$, where $h_{k,u,j}$ is the number of packets generated at time $k + j$ by the uth message (if any) arriving at time k. Therefore, $h_{k,u,j}$ can be viewed as a random impulse response function seen by the uth message. Hence, the number of packets to arrive at time n is given by

$$A_n = \sum_{k=-\infty}^{n} \sum_{u=1}^{X_k} h_{k,u,n-k}. \tag{4.1}$$

It is assumed that the one-dimensional processes $h_{k,u,\cdot} = (h_{k,u,j}\colon j \in Z_+)$ indexed by k and u are independent of each other and identically distributed. For brevity, below we write $E[h_j]$ for $E[h_{k,u,j}]$ and $\mathrm{cov}(h_j, h_{n+j})$ for $\mathrm{cov}(h_{k,u,j}, h_{k,u,n+j})$ (since these quantities do not depend on k or u) and $\mathrm{cov}(h_{u,j}, h_{v,n})$ for $\mathrm{cov}(h_{k,u,j}, h_{k,v,n})$ (which does not depend on k). An additional assumption is placed on (h_j) below in order to make analysis of the queue tractable.

The mean and autocorrelation functions of A_k can easily be found to be as follows.

$$\begin{aligned} E[A] &= E\left[\sum_{k=-\infty}^{n} \sum_{u=1}^{X_k} h_{k,u,n-k}\right] \\ &= E[X] \sum_{k=-\infty}^{n} E[h_{k,1,n-k}] \\ &= E[X] \sum_{j=0}^{\infty} E[h_j], \end{aligned} \tag{4.2}$$

$$\begin{aligned} R_A(n) = \sum_{j=0}^{\infty} &\left\{\mathrm{var}(X) E[h_j] E[h_{n+j}] + E[X]\mathrm{cov}(h_j, h_{n+j})\right\} \\ &+ E[X]^2 \left(\sum_{j=0}^{\infty} E[h_j]\right)^2 \end{aligned} \tag{4.3}$$

for $n \geq 0$. A derivation of (4.3) is given in the Appendix.

B. Queueing Analysis

Consider a discrete time $\cdot/D/1$ queue with unit service time, with the randomly filtered arrival process (A_k). Denote this queue by queue 1 and its mean queue length by N_1.

To derive N_1, we first construct another queue called queue 2 with exactly the same sequence of message initiations and same total number of packets associated with each message, but with the modification that all packets associated with a message arrive at the same time the message is initiated. Thus Y_k, the number of the packets to arrive at time k for queue 2, is given by

$$Y_k = \sum_{u=1}^{X_k} \sum_{j=0}^{\infty} h_{k,u,j}.$$

Let N_2 denote the mean queue length of queue 2. Further impose the following constraint on (h_j):

$$\Pr\left[\sum_{j=0}^{k} h_j \geq k + 1, \ \text{ for } 1 \leq k \leq L\right] = 1 \tag{4.4}$$

where $L = \max\{j: h_j \neq 0\}$ is the random length of the impulse function. Condition (4.4) ensures that whenever the service begins on a message in queue 1, the server is not idle until all packets of the message have been served. This implies that the departure processes of queue 1 and queue 2 are exactly the same. Therefore, $N_1 = N_2 - N_\triangle$, where $N_\triangle$ is the average number of packets which have arrived at queue 2 but which have yet to arrive at queue 1. At any time l, the number of such packets is

$$\sum_{k=-\infty}^{l} \sum_{u=1}^{X_k} \sum_{j=l+1-k}^{\infty} h_{k,u,j}$$

and its mean is

$$N_\triangle = E[X] \sum_{j=1}^{\infty} jE[h_j].$$

Since queue 2 is a discrete time $M_{\text{batch}}/D/1$ queue, its mean queue length N_2 is given by

$$N_2 = \frac{E[Y^2] - E[Y]}{2(1 - E[Y])}, \tag{4.5}$$

where

$$\begin{aligned} nE[Y] &= E\left[\sum_{u=1}^{X}\sum_{j=0}^{\infty} h_{u,j}\right] = E[X]\sum_{j=0}^{\infty} E[h_j] \\ E[Y^2] &= \sum_x \Pr\left[X = x\right] E\left[\left(\sum_{u=1}^{x}\sum_{j=0}^{\infty} h_{u,j}\right)^2 \middle| X = x\right] \\ &= E[X^2]E\left[\sum_{j=0}^{\infty} h_j\right]^2 + E[X]\text{var}\left(\sum_{j=0}^{\infty} h_j\right). \end{aligned} \tag{4.6}$$

C. A Special Model

Due to the complexity of (4.3), we consider a special model of the filter, with impulse function defined as

$$h_j = VI_{\{0 \le j \le L\}}$$

where V is a positive integer-valued random variable with mean $E[V]$ and second moment $E[V^2]$, L is a geometrically distributed random variable with parameter α, i.e., $\Pr[L = l] = \alpha(1-\alpha)^{l-1}$, for $l \ge 1$, and V and L are mutually independent. Hence, the mean and covariance function of (h_j) are, respectively

$$\begin{aligned} E[h_j] &= E[V] \Pr[L \ge j] = E[V](1-\alpha)^j, \\ \text{cov}(h_j, h_{n+j}) &= E[V^2](1-\alpha)^{n+j} - E[V]^2(1-\alpha)^{n+2j}. \end{aligned}$$

Then by (4.2),

$$E[A] = E[X]E[V]\sum_{j=0}^{\infty}(1-\alpha)^j = \frac{E[X]E[V]}{\alpha},$$

and by (4.2), the autocorrelation function can be found as

$$\begin{aligned} R_A(n) = (1-\alpha)^n &\left\{ \frac{E[X]E[V^2]}{\alpha} \right. \\ &\left. + \frac{E[V]^2(E[X^2] - E[X]^2 - E[X])}{1 - (1-\alpha)^2} \right\} \\ &+ \left(\frac{E[X]E[V]}{\alpha}\right)^2. \end{aligned} \tag{4.7}$$

The mean and second moment of the corresponding batch arrival process Y_k is

$$\begin{aligned} E[Y] &= \frac{E[X]E[V]}{\alpha} = E[A] \\ E[Y^2] &= \left(\frac{E[V]}{\alpha}\right)^2 E[X^2] \\ &\quad + E[X]\,\frac{(2-\alpha)E[V^2] - E[V]^2}{\alpha^2}. \end{aligned}$$

The mean number of arrivals at queue 2 yet to arrive at queue 1 is

$$N_\triangle = E[X]\sum_{j=1}^{\infty} jE[h_j] = E[X]E[V]\frac{1-\alpha}{\alpha^2}. \tag{4.8}$$

Therefore, the mean queue length N_1 is

$$N_1 = N_2 - N_\triangle = \frac{E[Y^2] - E[Y]}{2(1 - E[Y])} - \frac{1-\alpha}{\alpha^2}E[X]E[V]. \tag{4.9}$$

D. Matching Input Autocorrelation Functions

Given any $m > 0$, $\psi > 0$, and $0 < \lambda < 1$, (4.7) shows that there are processes obtained by random filtering using the special model of filter above, which have a specified autocorrelation function of the form

$$R_A(n) = m^2 + \psi\lambda^{|n|}. \tag{4.10}$$

All that is necessary is that the five parameters $E[X]$, $E[X^2]$, $E[V]$, $E[V^2]$, and α be selected so that (4.7) matches (4.10). This leaves two degrees of freedom in selecting five parameters, and infinitely many more degrees of freedom for selecting the distribution of A and V to match the selected first two moments. This yields a rich class of randomly filtered processes satisfying (4.10). Matching (4.7) with (4.10) yields

$$\begin{cases} \dfrac{E[X]E[V]}{\alpha} = m \\ 1 - \alpha = \lambda \\ \dfrac{E[X]E[V^2]}{\alpha} + \dfrac{E[V]^2(E[X^2] - E[X]^2 - E[X])}{1-(1-\alpha)^2} = \psi \end{cases} \tag{4.11}$$

and there are two constraints

$$E[X^2] > E[X]^2, \qquad E[V^2] > E[V]^2.$$

Choose $E[X]$ and $E[X^2]$ as free variables to yield

$$\begin{cases} \alpha = 1-\lambda \\ E[V] = \dfrac{\alpha m}{E[X]} \\ E[V^2] = \dfrac{(1-\lambda)\psi}{E[X]} - \dfrac{\alpha^2 m^2 (E[X^2]-E[X]^2-E[X])}{(1+\lambda)E[X]^3} \end{cases} \tag{4.12}$$

with a constraint

$$E[X^2] < \frac{(1+\lambda)E[X]^2\psi}{(1-\lambda)m^2} + E[X]^2 - \lambda E[X].$$

E. Queue Responses

Although there are infinitely many degrees of freedom for us to synthesize a fairly large class of processes, it turns out (much to our surprise) that the mean queue length for the filtered arrival processes with the special filter model described above is completely determined by the input mean and autocorrelation function in form of (4.10). To verify this, note that according to (4.9), N_1 can be expressed in terms of m, ψ, λ, and $E[Y^2]$ as

$$N_1 = \frac{E[Y^2]-m}{2(1-m)} - \frac{\lambda m}{1-\lambda}.$$

We can further show that $E[Y^2]$ is a function of the three parameters of the autocorrelation function as follows. By (4.8), $E[Y^2]$ can be rewritten as

$$E[Y^2] = \frac{E[V]^2(E[X^2]-E[X])}{\alpha^2} + \frac{2-\alpha}{\alpha^2}\, E[X]E[V^2].$$

Replace $E[X^2]-E[X]$ using the third equation of (4.11),

$$\begin{aligned} E[Y^2] &= \frac{1-(1-\alpha)^2}{\alpha^2}\left(\psi - \frac{E[X]E[V^2]}{\alpha}\right) \\ &\quad + \left(\frac{E[X]E[V]}{\alpha}\right)^2 + \frac{(2-\alpha)E[X]E[V^2]}{\alpha^2} \\ &= \frac{1-\lambda^2}{(1-\lambda)^2}\,\psi + m^2. \end{aligned}$$

Therefore, the mean queue length N_1 is

$$N_1 = \frac{\dfrac{1+\lambda}{1-\lambda}\,\psi + m^2 - m}{2(1-m)} - \frac{\lambda m}{1-\lambda}.$$

Thus, all processes of the form described in Section IV-C with the same autocorrelation function (4.10) *give rise to the same mean queue length.*

One then is led to wonder whether this result is true for the general class of randomly filtered arrival processes. It turns out that in general N_2 is uniquely determined by the input mean and autocorrelation function, because it can be shown (see the Appendix) that

$$E[Y^2] = \sum_{n=-\infty}^{\infty} \operatorname{cov}(A_0, A_n). \tag{4.13}$$

However, $N_\triangle = \sum_{j=1}^{\infty} jE[h_j]$ in general depends on a particular chosen model of h_j and does not appear to have a direct relationship with a given input autocorrelation function. Therefore, the queue responses to all possible randomly filtered processes are perhaps not completely determined by a general input mean and autocorrelation function, due to the variations in $N_\triangle$.

F. On the Marginal Distributions of the Processes of Section IV-C

We now briefly explore the one-dimensional marginal distributions of the random processes (A_k) under the conditions of Section IV-C, in order to better assess the breadth of statistical behavior within the model. First consider the case of short range dependence, or very little low frequency power, by taking $\lambda = 0$. Then $\alpha = 1$, and without loss of generality we also take $X \equiv 1$. The variables A_n are then simply any independent identically distributed Z_+ valued random variables with mean m and variance ψ. The fact that the mean queue length depends on the distribution of the A_n only through their mean and variance in this case is thus due to the surprising but well known Pollaczek–Khinchin type formula (4.5), showing that the mean delay for a discrete time $M_{batch}/D/1$ system (with D representing unit service times) depends only on the mean and variance of the batch sizes.

At the other extreme, consider moderate to long-range dependence (equivalently, large power at low frequencies) by taking λ to be close to one. Then for fixed n, A_n is the sum of a large number of independent integer random variables, each equal to zero with high probability. We also have $E[A_n] < 1$. Thus, the distribution of A_n is approximately that of a *compound* Poisson random variable. Recall that the compound Poisson distribution with parameters (μ, B) is the distribution of the sum of a random number of independent random variables, such that the number of random variables has the Poisson distribution of mean μ, and each of the random variables in the sum has distribution function B.

In fact, given any compound Poisson distribution on Z_+ with mean m and variance ψ and *any* λ with $0 \le \lambda < 1$, there is an arrival process generated by random filtering, with autocorrelation function (4.9) and marginal distribution equal to the given compound Poisson distribution.

Indeed, let (μ, B) denote the parameters of the given compound Poisson distribution. Then we use the model of Section IV-C and take $\alpha = 1-\lambda$, let X have the Poisson distribution with mean μ/a, and let V have distribution B. Then for $k \le n$, the number of messages initiated at time k which still can contribute packets at time n has the Poisson distribution with mean $(\mu/a)(1-a)^{n-k}$. Summing over k, we see that the number of messages generating packets at time n has the Poisson distribution with mean μ, and the number of packets contributed by each such message has distribution B. Thus, the process (A_n) produced has marginal distribution equal to the given compound Poisson distribution.

To summarize, we find that as λ increases from 0 to 1, the set of possible marginal distributions shrinks from the set of all distributions with mean m and variance ψ, down to the set of all compound Poisson distributions with mean m and variance ψ. Thus, for every λ, considerable variation in the marginal distribution is possible. Therefore, the complete determination of mean queue length by power spectrum that we observe for the processes of Section IV-C cannot be attributed to a lack of diversity in the one-dimensional marginal distributions. Rather we feel it is simply an extension of the invariance implicit in the well-known Pollaczek–Khinchin formula for mean delay.

V. Conclusions

This paper explores the variations of the mean queue length when different arrival processes with the same mean and autocorrelation functions are applied to a $\cdot/D/1$ queue. We observed that for two-state MMP processes, the mean queue length can vary quite substantially. Moderate variations in mean queue length were observed for the PSMP processes. Within the third class of processes explored, a subclass of randomly filtered white noise processes, we found that the mean queue length is completely determined by the input mean and autocorrelation. Of course arrival processes in practice can't be guaranteed to fall within the third class, so we conclude that the behavior of a queue *cannot* be predicted *solely* based on the mean and autocorrelation functions of its arrival processes.

This conclusion is consistent with recent trends in the literature. In particular, [3] demonstrated that the family of circulant modulated Poisson processes is sufficiently rich to match not only a given autocorrelation function in a large class, but also to approximately match the first-order marginal distribution of the input rate process. Tests run on data generated by MMP processes with a very large number of states, as well as real traffic sequences, showed that the combination of autocorrelation and first order distribution is quite effective. A similar conclusion was reported in [9], which used a first order discrete autoregressive model to match both the one-dimensional distribution and the one-frame correlation of real teleconference traffic.

The processes for which we found it most difficult to predict the queueing performance from the autocorrelation function alone were also the ones with the greatest power in the low frequency region. These are the two-state Markov processes (with both rates allowed nonzero) of Section II in the case that λ is near one. This is consistent with the observation of [2], to the effect that input power in the low frequency band is most essential to queueing analysis. If the input has a lot of power at low frequencies, then modifications of the low power components are sometimes possible for significantly changing the queueing performance. These observations show why, as is intuitively clear, the very long range dependence observed in some real data [10] might prove problematical in some situations, in spite of the successes reported in [3] and [9]. A promising approach for dealing with very long range dependence is to separate out the very slowly varying component of the traffic, as suggested in [4].

Finally, we found the spectral information did well at predicting the mean queue length for heavy traffic, which perhaps is to be expected since heavy traffic is the realm of diffusion approximations based on the central limit theorem.

In the remaining paragraphs of this paper, we discuss some potential applications of spectral analysis of datastreams for predicting queueing performance, and discuss the implications of this paper. We described in the introduction the appeal of using the power spectral density as an indication of the load imposed by a datastream. Datastreams are modeled as random sequences in this paper and the autocorrelation functions are defined by statistical expectation. In practice, the autocorrelation function can be associated to a datastream in different ways, depending on the context. For example, the associated power spectral density could simply be based on the history of datastreams of similar type. Or standard statistical spectral estimation methods based on empirical correlations could be applied to obtain the spectral estimates in real time.

An approach to provide quality of service guarantees for variable bit rate (VBR) applications in the asynchronous transfer mode (ATM) framework is mentioned in [4]. It was suggested that if a datastream has a large low frequency component, then the datastream should be allocated capacity at or near its peak requirement. This recommendation seems more prudent in light of our work. Not only does the presence of large power at low frequencies mean a datastream could have a large impact on the network performance, but it also makes the impact less predictable. Some other potential applications of analysis of queues via spectral methods are the following, listed in order of decreasing time scales.

- Variable rate source coders, such as video compression or compaction algorithms, should be designed to reduce the impact of the resulting datastreams on the networks that carry them. If a simple summary of such impact could be computed it could lead to the development of improved coders. The power spectrum of the coder output might be suggested as an appropriate measure.
- An approach for provisioning buffer sizes and transmission capacity in networks is to define a number of classes of traffic, each with an associated power spectrum (or bounds on the power spectra). We could then hope to predict the delay or loss probabilities based on the spectral information.
- A promising notion for quality of service guarantees and/or pricing for VBR traffic is that a contract be negotiated between the user and the network. Each could check for compliance. It was suggested in [1] that the contract involve the specification of the input power spectral density, or bounds on such. This is appealing since the power spectrum is easy to measure and gives a reasonable indication of network performance.
- In case of large propagation or measurement delay, effective rate control of available bit rate (ABR) datastreams in the ATM framework could make use of future predictions about the ABR queue length or delay. Such predictions could be based on predictions of the spectrum of load and excess capacity processes, obtained by statistical extrapolation methods (some of which are also based on spectral analysis of datastreams). Here the overall idea is to use prediction to enhance feedback control. In this application, estimates are not needed on a time scale longer than the reaction time of the rate adjustment algorithm.

The above applications are promising and deserve further consideration. At the same time, our results and the related results of [3] and [5] suggest that the power spectra alone is probably not adequate for such purposes in case the datastreams have a large component of power at low frequencies. In such cases, additional statistical information, such as the one-dimensional marginal distributions of datastreams, should be used.

APPENDIX

A. Demonstration that Autocorrelation Function Determines the Mean

If X is a wide sense stationary nonnegative random process, then the variance of the nth sample average satisfies

$$\mathrm{var}\left(\frac{\sum_{i=0}^{n-1} X_i}{n}\right) = \frac{1}{n}\sum_{|i|<n}\left(1-\frac{|i|}{n}\right)\mathrm{cov}(X_0,\, X_i).$$

If, in addition, said variance tends to zero as $n \to \infty$ (which means that the zero frequency component of X is deterministic) then the mean of X, $E[X_i]$, is determined from R_X as follows:

$$\begin{aligned} E[X_i]^2 &= \lim_{n\to\infty} E\left[\left(\frac{\sum_{i=0}^{n-1} X_i}{n}\right)^2\right] \\ &= \lim_{n\to\infty}\frac{1}{n}\sum_{|i|<n}\left(1-\frac{|i|}{n}\right)R_X(i). \end{aligned}$$

B. Derivation of (4.3)

$$\begin{aligned} \mathrm{cov}(A_0,\, A_n) &= \sum_{k=-\infty}^{0}\sum_{l=-\infty}^{n}\mathrm{cov}\left(\sum_{u=1}^{X_k} h_{k,\,u,\,-k},\, \sum_{v=1}^{X_l} h_{l,\,v,\,n-l}\right) \\ &= \sum_{j=0}^{\infty}\mathrm{cov}\left(\sum_{u=1}^{X_j} h_{u,\,j},\, \sum_{v=1}^{X_j} h_{v,\,n+j}\right). \end{aligned}$$

Since

$$\begin{aligned} &E\left[\sum_{u=1}^{X_j} h_{u,\,j}\cdot\sum_{v=1}^{X_j} h_{v,\,n+j}\right] \\ &\quad= \sum_x \Pr[X_j = x]\cdot E\left[\sum_{u=1}^{x} h_{u,\,j}\cdot\sum_{v=1}^{x} h_{v,\,n+j}\right] \\ &\quad= \sum_x \Pr[X_j = x]\cdot\{x^2 E[h_j]E[h_{n+j}] + x\,\mathrm{cov}(h_j,\, h_{n+j})\} \\ &\quad= E[X^2]E[h_j]E[h_{n+j}] + E[X]\,\mathrm{cov}(h_j,\, h_{n+j}) \end{aligned}$$

we have

$$\begin{aligned} &\mathrm{cov}\left(\sum_{u=1}^{X_j} h_{u,j},\, \sum_{v=1}^{X_j} h_{v,n+j}\right) \\ &\quad= E\left[\sum_{u=1}^{X_j} h_{u,j}\cdot\sum_{v=1}^{X_j} h_{v,n+j}\right] - E\left[\sum_{u=1}^{X_j} h_{u,j}\right]E\left[\sum_{v=1}^{X_j} h_{v,n+j}\right] \\ &\quad= E[X^2]E[h_j]E[h_{n+j}] + E[X]\mathrm{cov}(h_j,\, h_{n+j}) \\ &\qquad - E[X]^2 E[h_j]E[h_{n+j}] \\ &\quad= \mathrm{var}(X)E[h_j]E[h_{n+j}] + E[X]\mathrm{cov}(h_j, h_{n+j}). \end{aligned}$$

Therefore,

$$\begin{aligned} R(n) &= \mathrm{cov}(A_0,\, A_n) + E[A_0]E[A_n] \\ &= \sum_{j=0}^{\infty}\left[\mathrm{var}(X)E[h_j]E[h_{n+j}] + E[X]\,\mathrm{cov}(h_j,\, h_{n+j})\right] \\ &\quad + E[X]^2\left(\sum_{j=0}^{\infty}E[h_j]\right)^2. \end{aligned}$$

C. Derivation of (4.13)

$$\begin{aligned} &\sum_{n=-\infty}^{\infty}\mathrm{cov}(A_0,\, A_n) \\ &\quad= \mathrm{cov}(A_0,\, A_0) + 2\sum_{n=1}^{\infty}\mathrm{cov}(A_0,\, A_n) \\ &\quad= \sum_{j=0}^{\infty}\left\{\mathrm{var}(X)(E[h_j])^2 + E[X]\,\mathrm{var}(h_j)\right\} \\ &\qquad + 2\sum_{n=1}^{\infty}\sum_{j=0}^{\infty}\left\{\mathrm{var}(X)E[h_j]E[h_{n+j}]\right. \\ &\qquad\qquad \left. + E[X]\,\mathrm{cov}(h_j,\, h_{n+j})\right\} \\ &\quad= \sum_{j=0}^{\infty}\left\{\mathrm{var}(X)(E[h_j])^2 + E[X]\,\mathrm{cov}(h_j,\, h_j)\right. \\ &\qquad \left. + 2\sum_{l=j+1}^{\infty}\left[\mathrm{var}(X)E[h_j]E[h_l] + E[X]\,\mathrm{cov}(h_j,\, h_l)\right]\right\} \\ &\quad= \mathrm{var}(X)\left(\sum_{j=0}^{\infty}E[h_j]\right)^2 + E[X]\,\mathrm{var}\left(\sum_{j=0}^{\infty}h_j\right) \\ &\quad= \mathrm{var}(Y). \end{aligned}$$

REFERENCES

[1] S. Q. Li and C. L. Hwang, "Queue response to input correlation functions: Discrete spectral analysis," *IEEE/ACM Trans. Networking,* vol. 1, pp. 522–533, Oct. 1993

[2] ——, "Queue response to input correlation functions: Continuous spectral analysis," *IEEE/ACM Trans. Networking,* vol. 1, pp. 678–692, Dec. 1993

[3] C. L. Hwang and S. Q. Li, "On the convergence of traffic measurement and queueing analysis: A statistical-match queueing (SMAQ) tool," in *Proc. IEEE Infocom'95,* Apr. 1995, pp. 602–613.

[4] Y. Kim and S. Q. Li, "Timescale of interest in traffic measurement for link bandwidth allocation design," in *Proc. IEEE Infocom'96,* Mar. 1996, pp. 738–748.
[5] S.-Q. Li and J. D. Pruneski, "The linearity of low frequency traffic flow: An intrinsic I/O property in queueing systems," *IEEE/ACM Trans. Networking,* vol. 5, pp. 429–443, June 1997.
[6] M. F. Neuts, *Matrix Geometric Solutions in Stochastic Models.* Baltimore, MD: The Johns Hopkins Univ. Press, 1981.
[7] ——, *Structured Stochastic Matrices of the M/G/1 Type and Their Applications.* New York: Marcel-Dekker, 1989.
[8] J. N. Daigle, Y. Lee, and M. N. Magalhães, "Discrete time queues with phase dependent arrivals," *IEEE Trans. Commun.,* vol. 42, pp. 606–614, Feb.–Apr. 1994.
[9] A. Elwalid, D. Heyman, T. V. Laksman, D. Mitra, and A. Weiss, "Fundamental bounds and approximations for ATM multiplexers with applications to video conferencing," *IEEE J. Select. Areas Commun.,* pp. 1004–1016, Aug. 1995.
[10] W. E. Leland, M. Taqqu, W. Willinger, and D. V. Wilson, "On the self-similar nature of ethernet traffic (extended version)," *IEEE/ACM Trans. Networking,* vol. 2, pp. 1–15, Feb. 1994.

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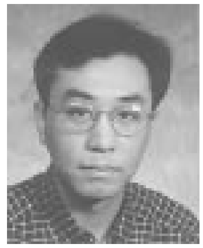

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