ADAPTIVE TRANSMISSION STRATEGIES AND ROUTING IN MOBILE RADIO NETWORKS

Bruce Hajek
Coordinated Science Laboratory
University of Illinois
1101 W. Springfield Avenue
Urbana, IL 61801

ABSTRACT

Local throughput in a mobile radio network is roughly defined as the rate at which packets are propagated in specified directions in local network regions. A key factor determining local throughput in an ALOHA or spatial TDMA network with randomly spaced stations is the transmission radius used by the stations. We demonstrate that allowing the transmission radius to depend on the desired direction of propagation can significantly increase local throughput.

The local throughput capabilities of a radio network can be effectively used only if adequate routing strategies are employed. This is illustrated by an example based on a symmetric demand assumption for stations uniformly distributed over a disc.

I. OPTIMAL FIXED RADIUS SELECTION

Suppose that the population of n stations is uniformly distributed within a circle of radius R. If n is large then in small regions the stations are distributed like a Poisson point process with intensity λ satisfying $n = \pi R^2 \lambda$. Assuming that the traffic demand is symmetric (i.e., uniformly distributed over all n(n-1) directed pairs of distinct stations) the mean distance between the source and destination of a packet is (see [4], or see [2] which is a chapter from [4])

$$\frac{128}{45\pi}\left(\frac{n}{\lambda\pi}\right)^{\frac{1}{2}} .$$

The network throughput in packet-hops for the ALOHA random access protocol can be approximated by

$$\frac{n}{\overline{A}_c \lambda e}$$

where A_c is the area covered by a transmission and $\overline{A}$ denotes the mean of A . (If the transmission radius is always a constant r then A_c is not random and is euqal to πr^2.) Thus, if $\overline{L}$ denotes the mean forward progress per successful transmission, the network end-to-end throughput in packets per time slot is

$$\gamma = \overline{L}\left(\frac{n}{\overline{A}_c \lambda e}\right)\Big/\left(\frac{128}{45\pi}\left(\frac{n}{\lambda\pi}\right)^{\frac{1}{2}}\right)$$

$$= (\eta/\sqrt{\lambda})(.72017)\sqrt{n} \qquad (1.1)$$

where

$$\eta = L/\overline{A}_c \quad . \qquad (1.2)$$

Since by equation (1.1) the end-to-end throughput is proportional to η we call η the <u>efficiency</u> of the transmission radius policy. The constant η is perhaps more meaningful than γ since it does not depend on the network's global geometry and is thus a "local" measure.

In [4] it was assumed that the transmission radius r is the same for all transmissions. The fixed radius r was chosen to maximize the efficiency η over all positive values. It was found that the optimum value of r is such that $\lambda A_c \approx 5.89$ - that is, the optimum fixed transmission radius is such that the mean number of stations within range of a given station is about six. This choice of transmission radius leads to efficiency

$$\eta_{opt,fixed\ r} \cong .135\ \lambda^{\frac{1}{2}} \quad . \qquad (1.3)$$

Using (1.1), this leads to the optimal network throughput $.0976\sqrt{n}$ reported in [4].

We are quick to remark that much of this analysis is heavily laden with approximations. For example, there is a problem in defining mean forward progress when no station is within range of the transmitter. (We avoid that particular problem in the next section.) See [4] for further discussion.

II. OPTIMAL ADAPTIVE TRANSMISSION RADIUS

Suppose that transmitters can vary their transmission radius with time (rather than using a fixed - although optimized - radius as in [4]), possibly as a function of the location of the other stations. How might the efficiency be improved?

First, we note that once a transmitter has identified an intended receiver for a packet transmission, it should use a transmission radius just large enough to reach that station. It remains to see which of the other stations should be the intended receiver.

Suppose a transmitter located at (0,0) must transmit a packet which is ultimately destined for a station with coordinates (z,0) where z is large. If a transmitter at (x,y) is chosen to receive the packet (and to then relay it on) the efficiency for that transmission would be

$$\eta(x,y) \triangleq \frac{x}{\pi(x^2+y^2)}$$

Clearly the adaptive transmission radius rule which maximizes η (defined in (1.2)) is to use the radius just large enough to reach the station whose coordinates (x,y) maximize $\eta(x,y)$ over all the stations. This rule is illustrated in Figures 1 and 2. Geometrically, one "scans" the region within a circle centered on the positive x axis which passes through (0,0). The diameter of the circle continuously increases until some station is contained in the region. That station becomes the intended receiver and the transmission radius used is the distance to that receiver. We will now compute the efficiency of this rule. All our analysis is under the assumption that n is so large that the distribution of stations located near the fixed station can be assumed to be Poisson with intensity λ per unit area.

Let A_s be the area of the region which is scanned. Then

$$P[A_s \geq c] = p[\text{no stations are in a given circle which has area } c] = e^{-\lambda c} \quad . \qquad (2.1)$$

Hence, A_s is exponentially distributed and $\bar{A}_s = \lambda^{-1}$. Since the diameter L of the region scanned is equal to $2(A_s/\pi)^{\frac{1}{2}}$ we have that

$$f_L(\ell) = \frac{\pi\lambda}{2}\exp(-\lambda\pi\ell^2/4) \text{ for } \ell \geq 0$$

and

$$\bar{L} = \lambda^{-\frac{1}{2}} \quad . \qquad (2.2)$$

Given that $L = \ell$, the coordinates (X,Y) of the designated receiver is a solution to

$$x^2 + y^2 = \ell x \qquad (2.3)$$

or

$$(x - \frac{\ell}{2})^2 + y^2 = \frac{\ell^2}{4} \quad .$$

For fixed x we compute from equation (2.3) that

$$\frac{dy}{d\ell} = \frac{x}{2y} = \frac{x}{2(\ell x - x^2)^{\frac{1}{2}}} \quad .$$

Now given that a region contains exactly one point of a Poisson point process with constant intensity, the point is uniformly distributed over the region. Applying this fact to the shaded region in Figure 3 yields that

$$f_{X|L}(x|\ell) = c_\ell \frac{dy}{d\ell}$$

where c_ℓ is chosen so that the conditional density integrates to one for ℓ fixed. To find c_ℓ we note that

$$\int_0^\ell \frac{dy}{d\ell}(x)\,dx = \frac{1}{2}\frac{dA_s}{d\ell} = \frac{1}{2}\frac{d\frac{\pi\ell^2}{4}}{d\ell} = \frac{\pi\ell}{4} \quad .$$

Therefore

$$f_{X|L}(x|\ell) = \frac{2x}{\pi\ell((\ell-x)x)^{\frac{1}{2}}} \text{ for } 0 \leq x \leq \ell \quad . \qquad (2.4)$$

From this we can compute that

$$E[X|L = \ell] = \int_0^\ell \frac{2x^2}{\pi\ell((\ell-x)x)^{\frac{1}{2}}}\,dx = \frac{3}{4}\ell \quad . \qquad (2.5)$$

Now using (2.2) and (2.5) we compute that

$$\bar{X} = E[E[X|L]] = \frac{3}{4}\bar{L} = \frac{3}{4}\lambda^{-\frac{1}{2}} \quad . \qquad (2.6)$$

Furthermore, the area of the region within transmission range is

$$A_c = \pi(X^2 + Y^2) = \pi LX$$

so that

$$\bar{A}_c = \pi\, E[LX] = E[LE[X|L]] = \frac{3\pi}{4}E[L^2] = 3\,E[\frac{\pi L^2}{4}] = 3\bar{A}_s = 3\lambda^{-1} \quad . \qquad (2.7)$$

From (2.6) and (2.7) we compute that the efficiency of the rule is

$$\eta_{opt} = \frac{\frac{3}{4}\lambda^{-\frac{1}{2}}}{3\lambda^{-1}} = .25\,\lambda^{\frac{1}{2}} \quad .$$

Comparing with equation (1.3) we see that by optimally adapting the transmission radius as a function of ultimate packet destination and other station locations, the efficiency can be increased by about 85%.

Equation (2.7) suggests that the optimal adaptive transmission radius policy we have chosen is such that on average, three stations will be in transmission range. (In analogy to [4], one might say that 3 is a magic number.) However, since the transmission range is random and is dependent on the station locations, the distribution of stations within transmission range is not conditionally Poisson. In fact, with probability one there is exactly one station within (actually, on the boundary of) the region scanned. The conditional distribution of stations in the unscanned region is Poisson, however.

Using (2.4) and the fact that $X = L\cos^2\Theta$ we easily derive that L and Θ are independent and

$$f_\Theta(\theta) = \frac{2\cos^2(\theta)}{\pi} \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \quad . \qquad (2.8)$$

Now for fixed θ the area of the region which is scanned but which is not within range of the transmission can be shown to equal

$$\Delta = \ell^2(\sin(2\theta) - 2\theta \cdot \cos(2\theta))/4$$

and using (2.8) the mean of this area is found (after elementary computations) to be

$$\bar{\Delta} = \left(\overline{L^2}\right)\left(\frac{17-\pi^2}{16\pi}\right) = \frac{17-\pi^2}{4\pi^2}\,\lambda^{-1} \quad .$$

Now the mean area of the region which is scanned and within the transmission range is $\lambda^{-1} - \bar{\Delta}$, and so the mean area of the unscanned portion of the region within transmission range is $2\lambda^{-1} + \Delta$ (see Fig. 4). Thus, the mean number of stations within transmission range, including

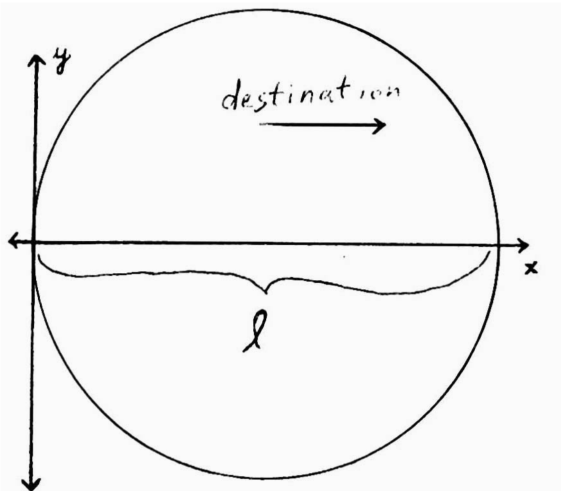

Figure 1. Locus of $\{(x,y): \frac{x}{x^2 + y^2} = \frac{1}{\ell}\}$ for ℓ fixed.

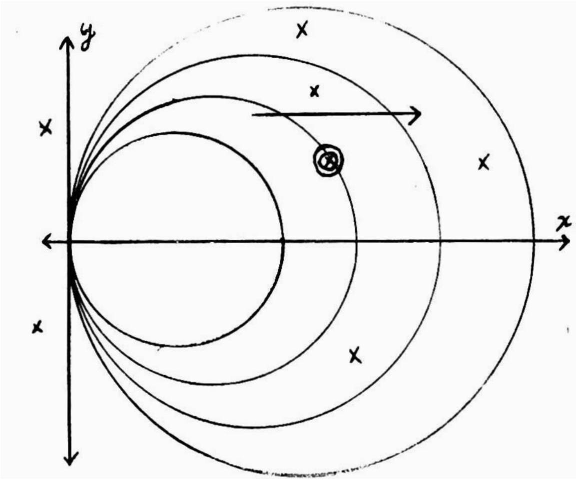

Figure 2. Illustration of selection rule - the preferred next receiver is circled.

the designated receiver, is

$$1 + (2 + \bar{\Delta}\lambda) \approx 3.18 \quad .$$

(One might say that 3.18 is a magic number.) It would be interesting (and it appears difficult) to find a dynamic transmission radius rule which minimizes the mean forward progress divided by the mean number of stations in transmission range, other than the intended receiver.

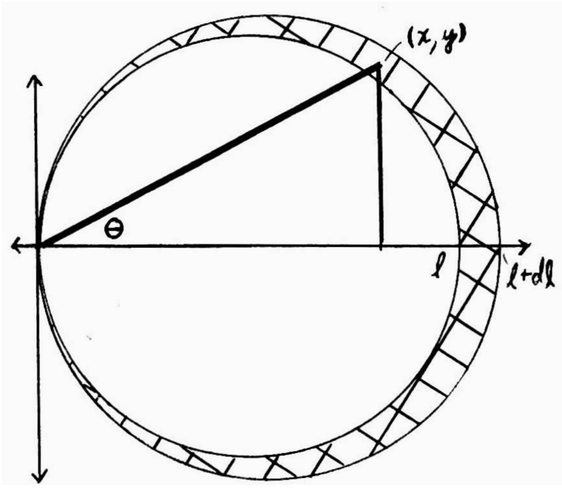

Figure 3. (X,Y) is uniformly distributed over shaded region given that $\ell \leqslant L \leqslant \ell + d\ell$.

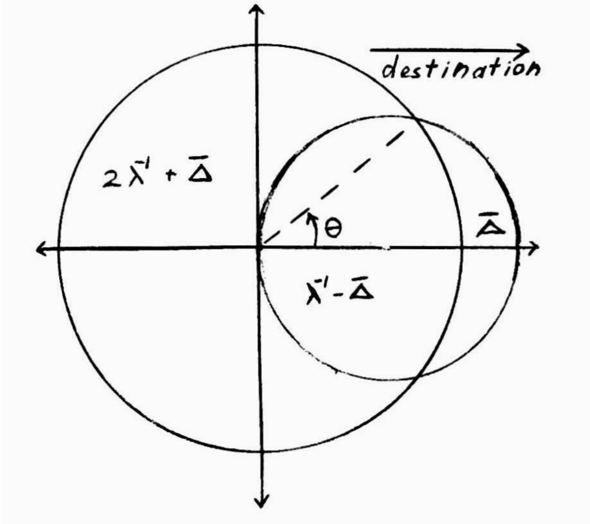

Figure 4. From left to right, the region unscanned and in transmission range, the region scanned and within range, and the region scanned and out of range are pictured and are labeled with their mean areas.

III. PRACTICAL CONCLUSIONS

It may not be feasible for stations to continuously vary their range from transmission to transmission. However, even if each station has only two available ranges, the same principles apply and thus using a destination dependent transmission range can increase throughput. Moreover, our study was motivated by the fact that in some situations the transmissions from a single station might be forced to have variable ranges. This situation is expected for the Navy's ITF Network due to the large variation in propagation characteristics over distinct frequency bands. It is important to emphasize that the variable ranges imposed by the environment can only be effectively exploited if packets can be transmitted at different frequencies for different hops along its path. That is, our results argue against the concept of independently running several networks, each in a different frequency band - rather they argue for a single network using all frequency bands simultaneously.

Another practical consideration is that it would most likely not be efficient (if even possible) for stations to execute the scanning procedure we suggested in order to determine the preferred receiver for a given ultimate destination. However, if a (yet to be developed) routing strategy is used which effectively incorporates station interdistance measurements, then we believe that routes will automatically be chosen which are roughly consistent with those determined by our rule. In this light, our calculations suggest that under effective routing strategies, the multiple access interference will be less (or else the throughput larger) than that predicted by [4].

The inpact of (physically) directed antennae on network considerations also deserves further study.

IV. GLOBAL NETWORK PERFORMANCE

The efficiency η we defined in Section I can be thought of as a measure of local throughput capacity. The formula (1.1) relates it to a global performance measure (end-to-end throughput) for a specific network topology and traffic demand. As noted in [4], in simulations, the end-to-end throughput was only a small fraction of that predicted by (1.1). In this section we indicate why indeed one should expect that the actual end-to-end throughput should be only a fraction of that predicted by equation (1.1) as long as a minimum hop routing rule is used.

Consider a continuum of stations uniformly distributed over a disc D of radius one. Suppose that the traffic demand is uniform and normalized so that the amount of traffic originating within one region and destined for another is equal to the product of the areas of the regions. The total rate that traffic enters the network is thus π^2.

Assume that "line-of-sight" routing is used. Then it makes sense to define the traffic load G(F) for a (Borel measureable) subset F of D by

$$G(F) = \int_{D\times D} m(F \cap \overline{uv})\,du\,dv \qquad (4.1)$$

where $\overline{uv}$ denotes the line segment with endpoints u and v, m denotes one-dimensional Lebesgue measure, and integration is over $D^2 \subset \mathbb{R}^4$ relative to the usual Lebesgue measure on $\mathbb{R}^4$. G(F) is the amount of packet movement which occurs within F and it has scale dimension packet×distance/time.

In the following proposition E(r) is the complete elliptic integral

$$E(r) = \int_0^{2\pi} (1-r^2\sin^2\theta)^{\frac{1}{2}}d\theta$$

$$= 2\pi\left[1-\left(\frac{1}{2}\right)^2 r^2-\left(\frac{1\cdot 3}{2\cdot 4}\right)^2\frac{r^4}{3}-\left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2\frac{r^6}{5}-\ldots\right].$$

Proposition. For $F \subset D$,

$$G(F) = \int_F g(|x|)dx \qquad (4.2)$$

where g is given by

$$g(r) = (1-r^2)E(r) \qquad (4.3)$$

The total load G(D) is

$$G(D) = 128\pi/45.$$

Proof. Introduce a point x on the line segment $\overline{uv}$. The segment $\overline{uv}$ together with x can be thought of as a marked directed line segment. It can be parameterized by the five-dimensional vector (u,v,a), where a denotes the distance of x from a (see Fig. 5.a) and then

$$G(F) = \int_{\{(u,v)\in D^2, x\in F\}} 1\ du\,dv\,da\ . \qquad (4.4)$$

The marked line segment can also be parameterized by (x,a,b,φ) where b is the distance between x and v, and φ is the measure of angle 0 x u (where 0 is the center of D, see Fig. 5.b). Using elementary computations which can be simplified using differential forms, the Jacobian determinant determined by the two coordinate systems can be found to be a + b (see [3, p. 46]). Thus Eq. (4.4) becomes

$$G(F) = \int_F\left\{\int_0^{2\pi}\int_0^A\int_0^B (a+b)\,da\,db\,d\varphi\right\} dx \qquad (4.5)$$

where for x and φ fixed, A and B denote the distance of x from the boundary of D along the directions at angle φ and at angle $\varphi + \pi$ from segment $\overline{Ox}$, rexpectively.

Now for x fixed,

$$\int_0^{2\pi}\int_0^{A}\int_0^{B}(a+b)\,da\,db\,d\varphi = \int_0^{2\pi}(A^2B+B^2A)\,d\varphi/2$$

$$= \int_0^{2\pi} A^2B\,d\varphi$$

$$= (1-|x|^2)\int_0^{2\pi} A\,d\varphi \quad (4.6)$$

where we use the fact that for any angle φ, $AB = 1 - |x|^2$. Finally, in view of Fig. 5.c,

$$\int_0^{2\pi} A\,d\varphi = \int_0^{2\pi} |x|\cos(\varphi) + (1-|x|^2\sin^2\varphi)^{\frac{1}{2}}\,d\varphi =$$

$$= \int_0^{2\pi} (1 - |x|^2\sin^2\varphi)^{\frac{1}{2}}\,d\varphi = E(|x|) \quad (4.7)$$

Combining Eqs. (4.5), (4.6) and (4.7) verifies Eq. (4.2) with g defined as in Eq. (4.3).

The net rate that traffic enters the network is π^2 and the mean path length is $128/(45\pi)$ [1, p. 41], [3]. Thus

$$G(D) = \pi^2\left(\frac{128}{45\pi}\right) = \frac{128\pi}{45} \qquad \square$$

The density profile g is shown in Fig. 6. Note that

$$\frac{\text{Peak traffic density (set } r=0)}{\text{Spatial average of traffic density}} = \frac{g(0)}{G(D)/\pi}$$

$$= \frac{45\pi}{64} \approx 2.20$$

Since maximum throughput is limited by the peak traffic density and since formula (1.1) is based on mean traffic density, we thus see that if line-of-sight routing (which is essentially dictated if n is large and minimum hop routing is used for the network considered in earlier sections) is used then the network throughput will be one third that predicted by (1.1). The high peak to average density ratio indicates the need for effective routing strategies which are not restricted to minimum hop routes.

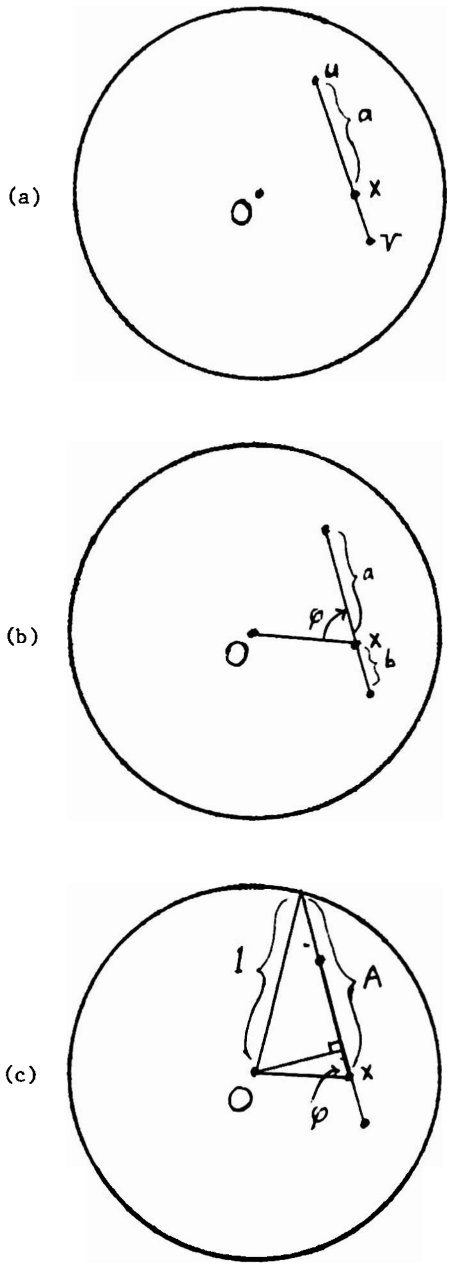

Figure 5. (a) and (b) represent two different parameterizations of a marked line segment in D. (c) shows A as a function of x and φ.

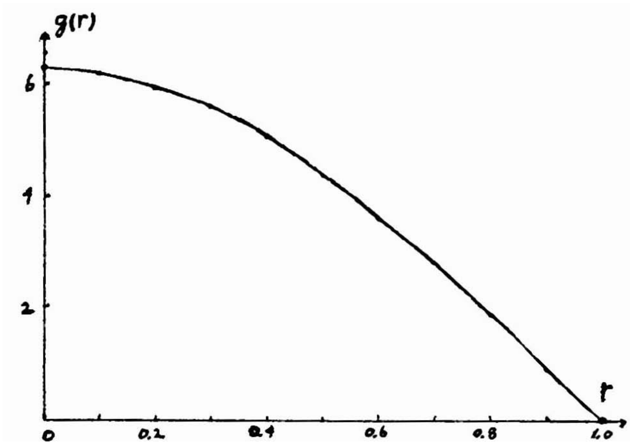

Figure 6. Traffic density vs. distance from center for uniform demand on a disk.

Acknowledgement: This research was supported by the Joint Services Electronics Program under contract N00014-79-C-0424 and by the Naval Research Lab under contract US NAVY N00014-80-C-0802.

[1] M. G. Kendall and P. A. P. Moran, Geometric Probability, Hafner Pub., New York, 1963.

[2] L. Kleinrock and J. Silvester, "Optimum transmission radii for packet radio networks or Why six is a magic number," Proceedings of the National Telecommunications Conference, pp. 4.3.1-5, 1978.

[3] L. A. Santalo, Integral Geometry and Geometric Probability, Addison-Wesley, Reading, MA, 1976.

[4] J. Silvester, "On the spatial capacity of packet radio networks," Ph.D. Dissertation, Computer Science Department, University of California, Los Angeles, March 1980.