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COMMUNICATION NETWORK THEORY

Doubly Randomized Protocols for a Random Multiple Access Channel with "Success–Nonsuccess" Feedback

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Abstract—We consider a model of a decentralized multiple access system with a nonstandard binary feedback where the empty and collision situations cannot be distinguished. We show that, like in the case of a ternary feedback, for any input rate $\lambda < e^{-1}$ there exists a "doubly randomized" adaptive transmission protocol which stabilizes the behavior of the system. We discuss also a number of related problems and formulate some hypotheses.

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1. INTRODUCTION

We consider a decentralized multiple access system model with an infinite number of users, a single transmission channel, and an adaptive transmission protocol; we consider a class of protocols where the user cannot observe the individual history of messages and the total number of messages. With any such a protocol, all users transmit their messages in time slot $[n, n+1)$ with equal probabilities p_n that depend on the history of feedback from the transmission channel.

Algorithms with ternary feedback "Empty–Success–Collision" were introduced in [1, 2]. It is assumed that the users can observe the channel output and distinguish among three possible situations: either no transmission ("Empty") or transmission from a single server ("Success") or a collision of messages from two or more users ("Conflict").

It is known since 1980s (see, e.g., [3,4]) that if the feedback is ternary, then the channel capacity is e^{-1}: if the input rate is below e^{-1}, then there is a stable transmission protocol, and if the input rate is above e^{-1}, then any transmission protocol is unstable. A stable protocol may be constructed recursively as follows: given probability p_n in time slot $[n, n+1)$ and a feedback at time $n+1$, probability p_{n+1} is greater than p_n if the slot $[n, n+1)$ was empty, $p_{n+1} = p_n$ if there was a successful transmission, and p_{n+1} is less than p_n if there is a conflict.

In [3] there was considered a multiplicative increase/decrease and assumed that the random number, ξ, of arrivals per typical slot has a finite exponential moment, $\mathbf{E}\, e^{c\xi} < \infty$, for some $c > 0$. In [4] there was considered an additive increase/decrease assuming the second moment $\mathbf{E}\, \xi^2$ to be finite. Later [5] it was shown that, without further assumptions on the input, the condition

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$\lambda = \mathbf{E}\,\xi < e^{-1}$ is sufficient for the existence of an multiplicative stable algorithm, and this result was extended onto a more general stationary ergodic input.

It is also known (see, e.g., [4,6,7]) that similar results (existence/nonexistence of a stable protocol if the input rate is below/above e^{-1}) hold for systems with either "Empty–Nonempty" binary feedback or "Conflict–Nonconflict" feedback.

In this paper we show that the channel capacity is again e^{-1} for a third type of the binary feedback which may be called "Success–Nonsuccess" feedback. The problem is interesting from a practical point of view, because in order for a receiver to distinguish between "collision" and "no transmission", it would have to differentiate between the increased energy or structure present when there is a collision of two or more packets, from thermal noise. That can be difficult or impossible for some receivers to do.

In [8,9] there was introduced and studied a model with the "success–failure" feedback, but with an extra option. There is a selected in advance station which works as follows. Given a nonsuccess feedback, the station may send, in the next time slot, a testing package to recognize what has happened, either empty slot or collision. Clearly, if an algorithm uses this option regularly, it is an algorithm with the ternary feedback which works slower that the conventional one (uses two time slots instead of one in the case of nonsuccess). In [8, 9], the authors introduce a class of algorithms that send a testing package from time to time only and show (numerically) that the lower is the rate of using this option, the closer to e^{-1} is the throughput.

In this paper, we do not allow this use of test packets. Our approach to the problem is to introduce a further (second) randomization. We consider a new class of "doubly randomized" protocols and show that, for any pair of numbers $0 < \lambda_0 < \lambda_1 < e^{-1}$, there exists a protocol from the class that makes stable a system with input rate λ, for any $\lambda \in [\lambda_0, \lambda_1]$. Then we formulate two conjectures on stability of other classes of protocols and, in particular of protocols that do not depend on an actual value of $\lambda \in (0, e^{-1})$. Our stability result is based on a generalized Foster criterion and the fluid approximation approach (see, e.g., [10]).

In a recent paper [11] (see also [12]), a stability result has been obtained for a similar model with "success–failure" feedback where a user may also take into account the arrival time of its message. It was shown that if the input rate is below 0.317, then there exist stable algorithms (called "algorithms with delayed intervals" in [11, 12]). The results of our paper are stronger in two directions: we show that a stable protocol exists if the input rate is below e^{-1} (clearly, $e^{-1} > 0.317$) and that there is no need to use the information about arrival times.

There is an interesting question which seems to be open: assume that we know arrival times of messages. Could this extra information increase actual channel capacity?

The paper is organized as follows. Section 2 contains the description of the model and of the class of transmission protocols under consideration, as well as the statement of the main result. Its proof is presented in Section 3. Then in Section 4 we introduce two more classes of protocols and formulate conjectures on their stability.

2. MODEL AND THE CLASS OF PROTOCOLS

We consider (a variant of) a multi-access system introduced in [13]. There is an infinite number of users and a single transmission channel available to all of them. Users exchange their messages using the channel. Time is slotted, and all message lengths are assumed to be equal to the slot length (say equal to one).

The input process of messages $\{\xi_n\}$ is assumed to be i.i.d., having a general distribution with finite mean $\lambda = \mathbf{E}\,\xi_1$; here ξ_n is the total number of messages arriving within time slot $[n, n+1)$ (for short, we call it "time slot n").

The systems operates according to an "adaptive ALOHA protocol" that may be described as follows. There is no coordination between the users, and at the beginning of time slot n each message present in the system is sent to the channel with probability p_n, independently of others. Thus, given that the total number of messages is N_n, the number B_n of those sent to the channel has conditionally the binomial distribution $B(N_n, p_n)$ (here $B_n \equiv 0$ if $N_n = 0$). Let $J_n = 1$ if $B_n = 1$ and $J_n = 0$ otherwise. If $J_n = 1$, then there is a successful transmission within time slot n. Otherwise, there is either an empty slot ($B_n = 0$) or a collision of messages ($B_n \geq 2$), so there is no transmission. Then the following recursion holds:

$$N_{n+1} = N_n - J_n + \xi_{n+1}. \tag{1}$$

A transmission protocol is determined by sequence $\{p_n\}$. We consider "decentralized" protocols: the numbers N_n, $n = 1, 2, \ldots$, are not observable, and only values of past J_k, $k < n$, are known. We consider protocols where $\{p_n\}$ are defined recursively in the Markovian fashion: p_n is a (random) number that depends on the history of the system only through p_{n-1} and J_{n-1}. Then a 2-dimensional sequence (N_n, p_n), $n = 1, 2, \ldots$, forms a time-homogeneous Markov chain.

In the paper we introduce three classes of decentralized protocols, prove a stability theorem for the first class, and conjecture similar results for the two others. To describe these protocols, we introduce additional notation.

Let $N_1 \geq 0$ be the initial number of messages in the system and $S_1 \geq 1$ a positive number (which is an "estimator" of unknown N_1). Let further $\beta \in (0, 1)$, $C > 0$, and $D > 0$ be three positive parameters, and let $\{I_n\}$ be an i.i.d. sequence that does not depend on the previous random variables, with $\mathbf{P}(I_n = 1) = 1 - \mathbf{P}(I_n = 0) = 1/2$.

Remark 1. In what follows, we assume a sequence of estimators $\{S_n\}$ to take integer values only, by assuming that D and C are integer-valued.

The class $\mathcal{A}_1$ of algorithms is determined by β, $C > 0$, $D > 0$, $\{J_n\}$ and $\{I_n\}$ as follows. The transmission probabilities p_n and the numbers S_n are updated recursively: given S_n, we let

$$p_n = \begin{cases} \beta/S_n & \text{if } I_n = 0, \\ 1/S_n & \text{if } I_n = 1. \end{cases}$$

Users, independently of each other, try to transmit their messages with the same probability p_n; after that, values of B_n and J_n are found, and then

$$S_{n+1} = \begin{cases} S_n + C & \text{if } J_n = 0, \\ S_n + CD & \text{if } J_n = 1 \text{ and } I_n = 0, \\ \max(S_n - CD, 1) & \text{if } J_n = 1 \text{ and } I_n = 1. \end{cases}$$

In words, the reason for the second randomization is as follows. When we get a successful transmission, we like keep our estimate S_n not far from the true value of N_n as long as possible. To increase our chances, we consider a randomized option for the probability p_n with taking two close but not identical values, β/S_n and $1/S_n$. Thus, if we get success, we increase the S-value for the smaller probability and decrease for the larger one.

We denote such an algorithm by $A_1(C, D, \beta) \in \mathcal{A}_1$. Two other classes of algorithms are defined in Section 4.

We can see that, with any algorithm introduced above, the sequence $\{(N_n, S_n)\}$ forms a time-homogeneous Markov chain. We introduce the following definitions.

Definition 1. We say that a Markov chain $\{(N_n, S_n)\}$ is *positive recurrent* if there exists a compact set $\mathcal{K} \in \mathbb{R}_+^2$ such that

- For any pair of initial values $(N_1, S_1) = (N, S)$,

$$\tau_{N,S} = \min\{n \geq 1: \ (N_n, S_n) \in \mathcal{K}\} < \infty \quad \text{a.s.};$$

- Furthermore,

$$\sup_{(N,S)\in\mathcal{K}} \mathbf{E}\, \tau_{N,S} < \infty.$$

A Markov chain $\{(N_n, S_n)\}$ is *Harris-ergodic* if there is a probability distribution μ such that, for any initial value $(N_1, S_1) = (N, S)$, the distributions of (N_n, S_n) converge to μ in the total variation as $n \to \infty$; i.e.,

$$\sup_A |\, \mathbf{P}((N_n, S_n) \in A) - \mu(A)| \to 0, \quad n \to \infty, \tag{2}$$

where the supremum is taken over all Borel sets A in $\mathbb{R}_+^2$.

It is well known (see, e.g., [14]) that a positive recurrent Markov chain $\{(N_n, S_n)\}$ is Harris-ergodic if it is aperiodic and there exist a positive integer m, probability measure φ, and positive number c such that

$$\mathbf{P}\big((N_m, S_m) \in \cdot \mid (N_1, S_1) = (n, s)\big) \geq c\varphi(\cdot) \tag{3}$$

for all $(n, s) \in \mathcal{K}$.

Definition 2. We say that a Markov chain $\{(N_n, S_n)\}$ is *transient* if there is an initial value $(N_1, S_1) = (N, S)$ such that $N_n + S_n \to \infty$ a.s., as $n \to \infty$.

Definition 3. Algorithm A is *stable* if the underlying Markov chain (N_n, S_n) determined by A is Harris-ergodic, and *unstable* if the underlying Markov chain is transient.

Here is our main result.

Theorem. *Let λ_0 and λ_1 with $0 < \lambda_0 < \lambda_1 < e^{-1}$ be any two numbers. There exist $C > 0$ and $\beta_1 \in (0, 1)$ such that, for any fixed $\beta \in (\beta_1, 1)$, there exists $D_0 = D_0(\lambda_0, \lambda_1, \beta)$ such that, for any $D \geq D_0$, algorithm $A_1(C, D, \beta)$ is stable for any input rate $\lambda \in [\lambda_0, \lambda_1]$.*

Remark 2. If $\lambda > e^{-1}$, then any algorithm with a binary feedback is unstable, since the same result is well known for any algorithm in a wider class of algorithms with ternary feedback (recall that we assume that at each time all users transmit with the same probability).

3. PROOF OF THEOREM 1

We need to introduce a number of auxiliary functions: for positive numbers β, λ, C, and D and for $0 \leq z < \infty$, let

$$j_1(z, \beta) = \frac{\beta z}{2} e^{-\beta z}, \qquad j_2(z) = \frac{z}{2} e^{-z}, \tag{4}$$

$$j(z, \beta) = j_1(z, \beta) + j_2(z), \tag{5}$$

$$a(z, \beta) = \lambda - j(z, \beta), \tag{6}$$

$$b(z, \beta) = C(1 - j(z, \beta)) + CD(j_1(z, \beta) - j_2(z)), \tag{7}$$

$$r(z, \beta) = a(z, \beta) - zb(z, \beta). \tag{8}$$

Clearly, for any $\beta > 0$ and as $z \to \infty$, $j_1(z, \beta)$, $j_2(z)$, and $j(z, \beta)$ tend to 0, and therefore $a(z, \beta) \to \lambda$, $b(z, \beta) \to C$, and $r(z, \beta) \to -\infty$.

We rely on the following auxiliary result.

Lemma. *The functions $j(z, \beta)$, $a(z, \beta)$, $b(z, \beta)$, and $r(z, \beta)$ satisfy the following conditions: for any $0 < \lambda_0 < \lambda_1 < e^{-1}$, there exists $\beta_1 \in (0, 1)$ such that, for any $\lambda \in [\lambda_0, \lambda_1]$, $\beta \in (\beta_1, 1)$, $C \geq C_1 := \dfrac{\lambda_1 + 1}{1 - e^{-1}}$, and $D \geq D(C)$ (where $D(C)$ is specified in the proof; see equality* (12) *below),*

- *The equation* $a(z,\beta)=0$ *has two roots* $0<z_1<z_2<\infty$;
- *The equation* $b(z,\beta)=0$ *has two roots* $0<t_1<t_2<\infty$;
- $0<t_1<z_1<t_2<z_2$;
- *The function* r *is continuous in* $z\in[0,\infty]$, $r(z,\beta)>0$ *for* $z\le z_1$ *and* $r(z,\beta)<0$ *for* $z\ge t_2$; *therefore,* $\inf\limits_{0\le z\le z_1} r(z,\beta)>0$, $\sup\limits_{t_2\le z\le\infty} r(z,\beta)<0$, *and all roots to the equation* $r(z,\beta)=0$ *lie in the interval* (z_1,t_2).

Proof. Introduce a further auxiliary function

$$b_1(z,\beta)=1-j(z,\beta)+D(j_1(z,\beta)-j_2(z)).$$

Then $b(z,\beta)=Cb_1(z,\beta)$.

We know that ze^{-z} (and also $j_2(z)$) is increasing in z if $z\in(0,1)$ and decreasing if $z>1$. Then, for $\beta<1$, $j_1(z,\beta)$ is increasing if $z<1/\beta$ and decreasing if $z>1/\beta$. Further, the function $m(\beta):=\min\limits_{1\le z\le 1/\beta} j(z,\beta)$ is strictly positive and tends to e^{-1} as $\beta\uparrow 1$.

For any $\lambda_1\in(0,e^{-1})$ and any $\varepsilon\in(0,e^{-1}-\lambda_1)$, one can choose $\widehat{\beta}_1<1$ so close to 1 that $m(\beta)\ge\lambda_1+\varepsilon$, for all $\beta\in[\widehat{\beta}_1,1)$. Then for any $\lambda\in(0,\lambda_1]$ and any $\beta\in[\widehat{\beta}_1,1]$, the equation $j(z,\beta)=\lambda$ has two roots, $z_1(\beta,\lambda)$ and $z_2(\beta,\lambda)$, with

$$z_1(\beta,\lambda)\le z_1(\beta,\lambda_1)<1<z_2(\beta,\lambda_1)\le z_2(\beta,\lambda).$$

By continuity of functions under consideration, for any $\lambda\le\lambda_1$ we have $\beta z_2(\beta,\lambda)\to z_2(1,\lambda)$ as $\beta\uparrow 1$, so one can choose $\beta_1\in[\widehat{\beta}_1,1)$ such that

$$\inf_{\lambda\in(0,\lambda_1]}\inf_{\beta\in[\beta_1,1]}\beta z_2(\beta,\lambda)>1.$$

Then, again for all $\lambda\in(0,\lambda_1]$ and all $\beta\in[\beta_1,1)$,

$$j_1(z_1,\beta)-j_2(z_1)<0<j_1(z_2,\beta)-j_2(z_2), \tag{9}$$

with $z_i=z_i(\beta,\lambda)$, for $i=1,2$.

Now we fix $\beta\in[\beta_1,1)$ and, for given $0<\lambda_0<\lambda_1$, let

$$D_0=\frac{2}{\inf\limits_{\lambda_0\le\lambda\le\lambda_1}(j_2(z_1)-j_1(z_1,\beta))}<\infty. \tag{10}$$

Then, for any $D\ge D_0$ and for any $\lambda\in[\lambda_0,\lambda_1]$, we have $b_1(z_1,\beta)<0$ and $b_1(z_2,\beta)>0$ (the latter inequality always holds).

Take any $D\ge D_0$, and let $t_1<t_2$ be the roots to the equation $b_1(z,\beta)=0$ (one can easily show that the latter equation has exactly two roots). Then, clearly, $0<t_1<z_1<t_2<z_2$.

Further, we may choose C such that all roots of the equation $r(z)=0$ lie in the interval (z_1,t_2). Indeed, for $z>z_2\ge 1$ and $\lambda\le\lambda_1$,

$$\begin{aligned}r(z,\beta)&=\lambda-j(z,\beta)-Cz(1-j(z,\beta))-CDz(j_1(z,\beta)-j_2(z))\\&\le\lambda_1-C(1-e^{-1})\le -1\end{aligned}$$

if

$$C\ge C_1:=\frac{\lambda_1+1}{1-e^{-1}}. \tag{11}$$

Further, for $z \le t_1 \le 1$ and $\lambda \ge \lambda_0$,

$$r(z,\beta) > \lambda - j(z,\beta) - Cz(1-j(z,\beta)) \ge \lambda - j(t_1,\beta) - Ct_1 \ge \lambda_0 - (1+C)t_1,$$

since $j(t_1,\beta) \le t_1 e^{-t_1} \le t_1$. The value of t_1 is monotonically decreasing to 0 as D tends to infinity. Therefore, one can choose $D_1 = D_1(C)$ such that $(1+C)t_1 \le \lambda_0/2$ for any $D \ge D_1$, and then let

$$D(C) = \max(D_0, D_1). \tag{12}$$

In more detail, since $\sup\limits_{t_0 \le t \le 1} \dfrac{1-j(t,\beta)}{j_2(t) - j_1(t,\beta)} < \infty$ for any $0 < t_0 < 1$ and $\lim\limits_{t \downarrow 0} \dfrac{1-j(t,\beta)}{j_2(t) - j_1(t,\beta)} = \infty$, we may choose $D_1 = D_1(C)$ so large that

$$t(D_1) = \max\left\{ t \in (0,1) : \frac{1-j(t,\beta)}{j_2(t) - j_1(t,\beta)} \ge D_1 \right\}$$

satisfies the inequality $t(D_1) \le \dfrac{\lambda_0}{2(C+1)}$. Thus, for any $D \ge D_1$, we have $t_1 < t(D_1) \le \dfrac{\lambda_0}{2(C+1)}$.

It remains to note that $r(z,\beta) > 0$ for $z \in (t_1, z_1]$, since $a(z,\beta) \ge 0$ and $b(z,\beta) < 0$, and that $r(z,\beta) < 0$ for $z \in [t_2, z_2)$, since $a(z,\beta) < 0$ and $b(z,\beta) \ge 0$. △

Now, for the proof of the theorem, we apply the lemma as follows.

We fix β and write for short $j(z)$, $a(z)$, $b(z)$, and $r(z)$ instead of $j(z,\beta)$, $a(z,\beta)$, $b(z,\beta)$, and $r(z,\beta)$.

Step 1. We introduce fluid limits for the Markov chain under consideration. Let the Markov chain $(N_k^{m_0}, S_k^{s_0})$ start from initial values $N_0 = m_0$ and $S_0 = s_0$, and assume that $v_0 := m_0 + s_0 \to \infty$ and that $m_0/v_0 \to x_0$, $s_0/v_0 \to y_0$, where $x_0 + y_0 = 1$ and $x_0, y_0 \ge 0$. Consider a continuous-time Markov process $(N^m(t), S^s(t))$, where

$$N^m(t) = N^m_{[tv]}, \qquad S^s(t) = S^s_{[tv]}$$

(here $[z]$ is the integer part of a number z). Then we consider a family of weak limits of these processes, as $v \to \infty$. They are indexed by their initial value (x_0, y_0) with $x_0 \ge 0$, $y_0 \ge 0$, and $x_0 + y_0 = 1$. By following the standard scheme (see, e.g., [15–17]), one can easily show that each such a limit, say $(\widetilde{N}(t), \widetilde{S}(t))$, is a Lipschitz function with continuous derivatives, and its derivatives are the functions $a(z)$ and $b(z)$ that were introduced above. In more detail, for any $x_0, y_0 \ge 0$, $x_0 + y_0 = 1$, for any fluid limit $(\widetilde{N}(t), \widetilde{S}(t))$, $t \ge 0$, that starts from the initial value $\widetilde{N}(0) = x_0$, $\widetilde{S}(0) = y_0$, and for any time $t \ge 0$, we let $z = z(t) = \widetilde{N}(t)/\widetilde{S}(t) \in [0,\infty]$ if $\widetilde{N}(t) + \widetilde{S}(t) > 0$. Then the derivatives are $d\widetilde{N}(t)/dt = a(z)$ and $d\widetilde{S}(t)/dt = b(z)$.

To see that the derivatives are $a(z)$ and $b(z)$ indeed, we may find the one-step drift. We have

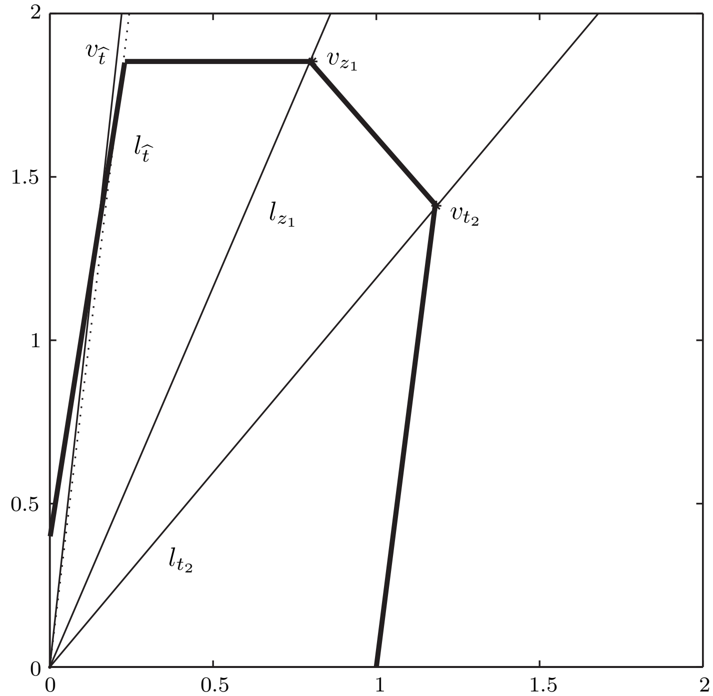
$$j(m,s) := \mathbf{E}(J_n \mid N_n = m,\, S_n = s) = \frac{m\beta}{2s}\left(1 - \frac{\beta}{s}\right)^{m-1} + \frac{m}{2s}\left(1 - \frac{1}{s}\right)^{m-1}.$$

Then

$$a(m,s) := \mathbf{E}(N_{n+1} - N_n \mid N_n = m,\, S_n = s) = \lambda - j(m,s).$$

In the conditions of the theorem and for $s > CD$,

$$\begin{aligned} b(m,s) &:= \mathbf{E}(S_{n+1} - S_n \mid N_n = m,\, S_n = s) \\ &= C(1 - j(m,s)) + CD\left(\frac{m\beta}{2s}\left(1 - \frac{\beta}{s}\right)^{m-1} - \frac{m}{2s}\left(1 - \frac{1}{s}\right)^{m-1}\right). \end{aligned}$$



Graph $L(x, y) = 1$ in the case $\lambda = 0.1$, $\beta = 0.98$, $C = 2.1$, and $D = 10000$

Then, as $m + s \to \infty$, $m/s \to z$,

$$j(m, s) \to j(z), \qquad a(m, s) \to a(z), \qquad b(m, s) \to b(z).$$

Step 2. We have to show next that any fluid limit $(\widetilde{N}(t), \widetilde{S}(t))$ (that starts from the initial value $(\widetilde{N}(0), \widetilde{S}(0))$ with $\widetilde{N}(0) + \widetilde{S}(0) = 1$) is *stable* in the following sense: for some $\varepsilon \in (0, 1)$, there exists finite time t_ε such that $\widetilde{N}(t_\varepsilon) + \widetilde{S}(t_\varepsilon) \le 1 - \varepsilon$. Then, by the general theory (see, e.g., [17]), the positive recurrence of the underlying Markov chains follows.

In the positive quadrant $\mathbb{R}^2 \setminus \{(0, 0)\} = \{(x, y) : \ x, y \ge 0, \ x + y > 0\}$, introduce a vector field of "rates": the rate from point (x, y) is $(a(z), b(z))$, where $z = x/y$. From the lemma and Step 1, we may deduce that this vector field is "self-similar" (rates do not change along any line with tangent z that starts from the origin). Functions $(a(z), b(z))$ are continuous in $z \in [0, \infty]$. Since $a(0) > 0$, $b(0) > 0$, $a(\infty) > 0$, $b(\infty) > 0$, and the functions change their signs in the "right" order $0 < t_1 < z_1 < t_2 < z_2$, we have, in particular, $a(z) < 0$ and $b(z) < 0$ for $z \in (z_1, t_2)$, and $\inf_z(|a(z)| + |b(z)|) > 0$. Each trajectory of a fluid limit (fluid trajectory, for short) is a "geodesic" line with respect to this vector field.

From the properties of the function r proved in the lemma, it follows that any fluid trajectory moves towards the cone $\mathcal{C} := \{(x, y) : \ x/y \in [z_1, t_2]\}$, hits the cone at some time instant, and then never leaves it and drifts towards the origin. To make this observation more rigorous, we introduce a positive function $R(z)$ with the following properties: $R(0) = 1$, and the pair $(\arctan(z), R(z))$, $0 \le z \le \infty$, represents a graph (in polar coordinates) of a smooth function that splits the positive quadrant into two domains (where one of the domains is a convex compact neighborhood of the origin).

Then, for any constant $c > 0$, we draw a line $(\arctan(z), cR(z))$ and introduce a test function $L(x, y)$ by letting $L(x, y) = c$ if the point (x, y) belongs to the line $(\arctan(z), cR(z))$. Further, the line is such that, for c large enough, the vector $(a(z), b(z))$ is directed into the compact domain, and its normal component to the line is not less than a certain positive value.

The construction follows a number of routine steps, therefore we provide a sketch of the proof and a clarifying figure only.

First we construct a continuous piecewise-linear function. Then we make it smooth around the points where the line changes it direction. The piecewise linear function is introduced as follows. We start from a fixed point, say $(1, 0)$, on the abscissa axis. Since $\lambda/C < 1 < t_2$, we may choose an angle slightly larger than $\arctan(\lambda/C)$, say $\varphi \in (\arctan(\lambda/C), \pi/4)$, and draw a straight line from $(1, 0)$ at angle φ (measured from the ordinate axis). It intersects the line l_{t_2} with tangent t_2 that starts from the origin (again we measure the tangent with respect to the ordinate axis) at some point, say v_{t_2}. From this point, we draw another straight line at the angle of $-\pi/4$ until it crosses the line l_{z_1} with tangent z_1 (that starts from the origin) at some point, say v_{z_1}.

Now we recall that $\lambda/C > t_1$; see the proof of the lemma. Therefore, one can take any $\varepsilon \in (0, (\lambda/C - t_1)/2)$, let $\widehat{t} = t_1 + \varepsilon$, and draw a line with tangent $\widehat{t}$ from the ordinate axis that starts from the origin. Starting from point v_{z_1}, we draw a horizontal line in the left direction, and it intersects with $l_{\widehat{t}}$ at some point, say $v_{\widehat{t}}$. Finally, starting from $v_{\widehat{t}}$, we draw a straight line at the angle $\arctan(\lambda/C - \varepsilon)$ in the left-down direction until it crosses the ordinate line, say at point v_0.

Therefore, we have drawn a continuous and convex piecewise linear line. Then we make it smooth (say differentiable) by changing in small neighborhoods of the corners (around the points v_{t_2}, v_{z_1}, and $v_{\widehat{t}}$), with keeping it convex. This completes the construction of the line $L(x, y) = 1$.

Recall that all other lines $L(x, y) = c$ are obtained by scaling. Then, using routine calculations and the properties of the function r, one can show that, for c large enough and for each $z \in [0, \infty]$, the drift vector $(a(z), b(z))$ is directed into the compact domain $\{(x, y) : L(x, y) \le c\}$, and its normal projection is uniformly positive for all $z \in [0, \infty]$. This implies positive recurrence of the underlying Markov chain.

To conclude that Harris conditions hold, we observe that the compact domain contains only finitely many states (since N_n and S_n are integer-valued), that all these states intercommunicate, and that the Markov chain is aperiodic since $\mathbf{P}(\xi_1 = 0) > 0$. This completes the proof of the theorem.

4. CONJECTURES

Here we introduce two more classes of transmission protocols and conjecture corresponding stability results.

Let $\mathcal{H}$ be a class of functions $h\colon [1, \infty) \to [0, \infty)$ such that $h(1) = 0$, $h(x) \uparrow \infty$ is nondecreasing in x, $x - h(x) \uparrow \infty$ is nondecreasing in x, and $h(x)/x \to 0$ as $x \to \infty$. With each $h \in \mathcal{H}$, we associate a class $\mathcal{E}_h$ of positive functions $\varepsilon_h\colon [1, \infty) \to (0, 1/2]$ such that $\varepsilon_h(x) \to 0$ and $h(x)\varepsilon^2(x) \to \infty$ as $x \to \infty$.

The two other classes of algorithms differ from the first class in the following.

Algorithms from the second class $\mathcal{A}_2$ differ from those from the class $\mathcal{A}_1$ only in a way the S_n are updated: the constant CD is replaced by a function $h \in \mathcal{H}$. More precisely, the algorithms are determined by parameters β and C, function $h(x)$, and random variables $\{J_n\}$ and $\{I_n\}$. Given S_n, we again let

$$p_n = \begin{cases} \beta/S_n & \text{if } I_n = 0, \\ 1/S_n & \text{if } I_n = 1, \end{cases}$$

but now define S_{n+1} by

$$S_{n+1} = \begin{cases} S_n + C & \text{if } J_n = 0, \\ S_n + h(S_n) & \text{if } J_n = 1 \text{ and } I_n = 0, \\ \max(S_n - h(S_n), 1) & \text{if } J_n = 1 \text{ and } I_n = 1. \end{cases}$$

For such algorithms, we use the notation $A_2(C, h, \beta) \in \mathcal{A}_2$.

We further modify the class $\mathcal{A}_2$ as follows: we replace β by $1 - \varepsilon_h$; this will form the third class $\mathcal{A}_3$. More precisely, the algorithms are determined by C, h, ε_h, $\{J_n\}$, and $\{I_n\}$. Given S_n, we now let

$$p_n = \begin{cases} (1 - \varepsilon_h(S_n))/S_n & \text{if } I_n = 0, \\ 1/S_n & \text{if } I_n = 1, \end{cases}$$

and then define S_{n+1} as for algorithms of the second class:

$$S_{n+1} = \begin{cases} S_n + C & \text{if } J_n = 0, \\ S_n + h_(S_n) & \text{if } J_n = 1 \text{ and } I_n = 0, \\ \max(S_n - h(S_n), 1) & \text{if } J_n = 1 \text{ and } I_n = 1. \end{cases}$$

For such protocols, we use the notation $A_3(C, h, 1 - \varepsilon_h) \in \mathcal{A}_3$.

We can see that again, with any algorithm from the classes $\mathcal{A}_2$ or $\mathcal{A}_3$, a sequence $\{(N_n, S_n)\}$ forms a time-homogeneous Markov chain.

We believe the following two statements be true.

Conjecture 1. *Let $0 < \lambda_0 < e^{-1}$ be any number. There exists $C > 0$ and $\beta_2 \in (0, 1)$ such that, with any $\beta \in (\beta_2, 1)$ and some function $h \in \mathcal{H}$, algorithm $A_2(C, h, \beta)$ stabilizes the system, for any input rate $\lambda < \lambda_0$. If, on the contrary, either $\beta < \beta_2$ or $\lambda > \lambda_0$, then the algorithm $A_2(C, h, \beta)$ is unstable in the system with input rate λ, for any $h \in \mathcal{H}$.*

Conjecture 2. *Any algorithm $A_3(C, h, 1 - \varepsilon_h)$ from the third class stabilizes the system, for any input rate $\lambda < e^{-1}$.*

Remark 3. The conjectures may hold for a broader classes of algorithms if one assumes that, in the recursion for S_n, function h is replaced by two functions, h_1 in the second line and h_2 in the third.

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