On the theory of combinatorial auctions and the sale of wireless spectrum

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Outline

1. FCC Standard Auction 73, focus on blocks A,B,D,E
2. Ascending price auction theory, complements and substitutes
3. FCC Hierarchical Auction 73, focus on block C
4. Example illustrating effect of package auction
5. Discussion: choices and goals for FCC auctions

1. Standard FCC Auction (Auction 73, blocks A,B,D,E)

Auction of 700 MHz band licenses in USA
1,099 licenses offered in Auction 73.
January-March 2008 Releasing old TV spectrum

All blocks in 700-800 MHz band:

Block A:12MHz 176 licenses Reserve price $1.8B
Block B:12MHz 734 licenses Reserve price $1.3B
Block D:10MHz 1 license* Reserve price $1.3B
Block E:6 MHz 176 licenses (unpaired spectrum)
Reserve price $0.9B

(Block D is subject to public/private partnership agreement)

Part 1: Standard FCC Auction (Auction 73, blocks A,B,D,E)

Each license: -number of bidding units = upfront payment,
-min opening bid

044-A Knoxville TN, pop. 983,000, 492K bidding units
min opening bid: $720K

068-A Champaign-Urbana, pop. 631,000, 292K bidding units
min opening bid: $227K

Part 1: Standard FCC Auction (Auction 73, blocks A,B,D,E)

Prior to auction, bidders file Form 175 to:

establish eligibility to bid for a set of licenses

deposit upfront cost for each license declared.

044-A Knoxville TN,	492K bidding units
068-A Champaign-Urbana, IL,	292K bidding units
075-B Austin, TX	696K bidding units
003-E Boston region	2.2M bidding units
Block D Nationwide	128M bidding units

Part 1: Standard FCC Auction (Auction 73, blocks A,B,D,E)

Bidding proceeds in rounds. Bidders place bids on licenses. A *provisional winner* is selected for each license from among the highest bidders.

-- Ties are broken randomly.

-- Carry over previous winner if no new bids.

Part 1: Standard FCC Auction (Auction 73, blocks A,B,D,E)

Anonymnity

The identities/owners of the bidders are released before the bidding begins. *No other info is released:*

e.g. the set of licenses registered for is not revealed.

the number of bidders for each license isn't revealed

the identity of provisionally highest bidder in each round isn't revealed

Bidder are forbidden from communicating with other bidders for same licenses (anti-collusion rules) about bids.

Part 1: Standard FCC Auction (Auction 73, blocks A,B,D,E)

Bid amounts: Multiple choices (no numbers entered--minimizes errors and signaling)

Smallest choice: equal to min opening bid if not bid on before, or equal to 10% to 20% more than previous provisionally winning bid (percentage increase depends on bidding activity for license). But smallest choice won't exceed $100M over previous winning bid.

Larger choices in fixed increments (e.g. 10% increments).

Part 1: Standard FCC Auction (Auction 73, blocks A,B,D,E)

Activity rule ("use it or lose it"). Bidders must use at least 80% of their eligibility (based on bidding units) in each round, or reduce eligibility.

Percent increases to 95% later in auction "Stage 2"

Part 1: Standard FCC Auction (Auction 73, blocks A,B,D,E)

Exposure problem:

Bidder may want to acquire a bundle of licenses, but may have little interest in a partial bundle.
i.e. *complementary* licenses

Bidder may find it impossible to win, but get stuck with a partial package.

Early rounds offer price discovery--reduces chance of exposure.

Part 1: Standard FCC Auction (Auction 73, blocks A,B,D,E)

Withdrawal rule: A bidder, on a one time basis, can withdraw a set of provisionally winning bids.
-Bidder is liable for possible loss in revenue.

e.g. if bidder is a provisional winner at $120M, withdraws, and license ends up being sold for $110M, then bidder pays $10M penalty.

Part 1: Standard FCC Auction (Auction 73, blocks A,B,D,E)

Auction ends when no new bids offered in a round. If the sum of provisionally winning bids for all licenses in a block exceed the reserve price for block, bids become winners.

Auction 73 outcome: Bidding took 38 days for xxx rounds. 214 bidders participated, 101 bidders won 1090 licenses. AT&T gained nationwide coverage in Block B. Verizon won nationwide coverage block C (described next). Blocks A,B,E made reserve bids, raised around $14B. Block D (nationwide, public safety restrictions) did not make $1.5B reserve bid.

Part 1: Standard FCC Auction (Auction 73, blocks A,B,D,E)

2. Ascending price auction theory, complements and substitutes

Some notions of when goods are substitutes for a given buyer:

Subadditivity: $v(A \cup B) \leq v(A) + v(B)$

Submodularity: $v(A \cup B) + v(A \cap B) \leq v(A) + v(B)$

Substitutes property:

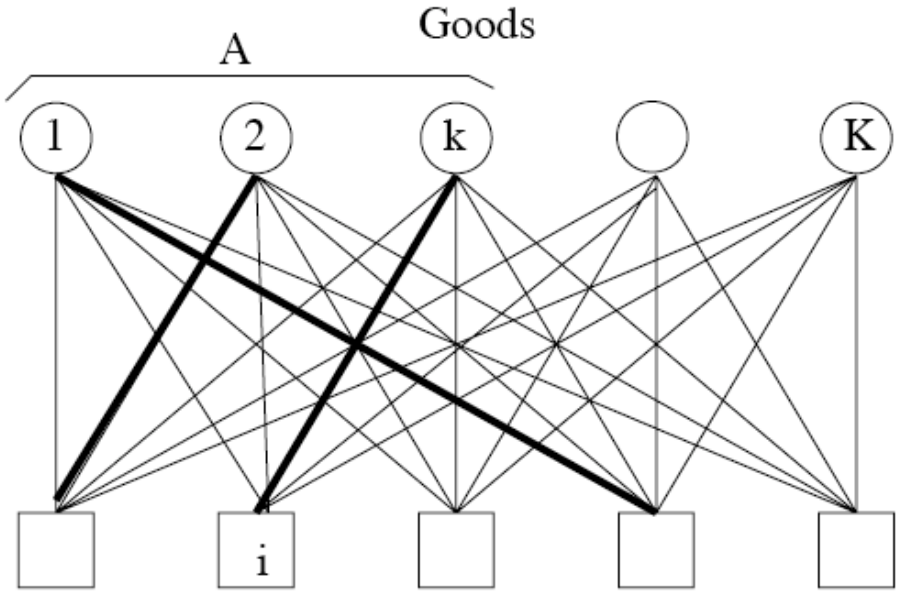
Substitutes property:

Consider the bundle of goods demanded by a bidder as a function of a vector of prices.

Definition: The bidder valuation function satisfies the *substitutes condition* if when prices are increased on some goods, goods in demanded bundle with no price increase are in a new demand bundle (for the higher prices).

Insures ascending price auction leads to efficient allocatio

Example of a presentation algorithm:



2. Ascending price auction theory, complements and substitutes

My recent paper: Give algorithm to generate functions satisfying substitute property by Monte Carlo simulation. Show that combinatorial presentation algorithm covering all substitute valuations on K items must take input list of length $2^{(1-\epsilon)K}$ from bidder.

2. Ascending price auction theory, complements and substitutes

3. FCC Hierarchical Auction 73, focus on block C

Block C consists 22MHz of spectrum, broken into 12 regions

1-6 six regions in lower 48 states
7 Alaska
8 Hawaii

10 Puerto Rico, US Virgin Islands
12 Gulf of Mexico

9 Guam, Northern Mariana Islands
11 American Somoa

Also grouped int
three packages

Bidders could bid on individual licenses and/or any of the three packages.
Provisional winners are for individual licenses if sum of individual bids exceeds largest package bid.
Else for package bid.
Requires extension/modification of eligibility rules, and minimum bids. (Fairly straight forward.)

3. FCC Hierarchical Auction 73, focus on block C

For example, if a bid for a package is provisional winner, the minimum acceptable bid for individual licenses are equal to bidding units (initially assigned) scaled up so sum exceeds provisionally winning bid for package by 10-20%.

3. FCC Hierarchical Auction 73, focus on block C

Q. When could the package option make a difference?

A. One strong bidder desiring nationwide C band license, but much less interested in part of nation versus possibly a second, very determined bidder, who wants to cover half of nation. First bidder can bid on package aggressively without fearing exposure problem.

3. FCC Hierarchical Auction 73, focus on block C

Total reserve price for block C: $4.5B
Due to influence from Google, if sold in Auction 73,
block C would have open platform, open content restrictio

Result: Verizon bought C: 1-6 separately for about $5B

3. FCC Hierarchical Auction 73, focus on block C

4. Example illustrating effect of package bidding

Example scenario

Bidder 1: values A at $1M, B at $1M, A+B at $2M
Bidder 2: values A or B or A+B at $0.8M
Reserve prices for A and B: $0.3M

Three scenarios:
Ascending auction price-taking
Bidder 1 gets both for $1.6M
Ascending auction Bidder 1 strategic
Bidder 1 gets A for $0.3M
Package biding, bidder 1 stragtegic
Bidder 1 gets both for $0.8M

5. Discussion: choices and goals for FCC auctions

Some design choices:

What spectrum to auction and when. (e.g. other blocks of 700MHz band were sold earlier)

Granularity: Why 1099 licenses? Why groupings?

Auction format

Choice of bidding units, minimum bids, reserve prices

Possible design goals:

- Revenue maximization? What time horizon? What revenue?

- Efficiency (sell to bidders who have highest value for spectrum)?

- Balance interests of encumbents vs. new entrants

- Low complexity of communication and/or computation
  - -for autioneer
  - -for buyers--including burden of determining good bids

5. Discussion: choices and goals for FCC auctions

REFERENCES

FCC auction website [primary source for this talk]

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Thanks!