Computational Lower Bounds for Community Detection on Random Graphs

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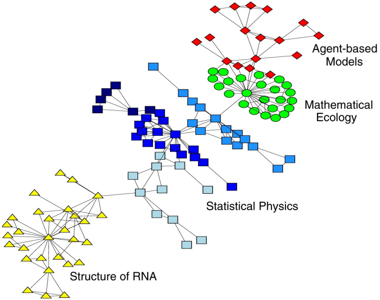
Community detection in networks

- Networks with community structures arise in many applications



Collaboration network: 118 scientists [Girvan-Newman ’02]

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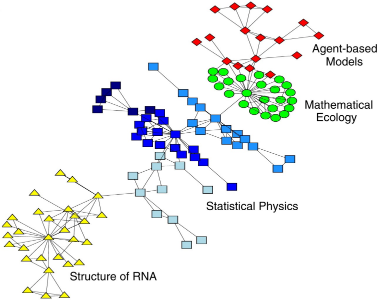

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- Task: Find underlying communities based on the network topology

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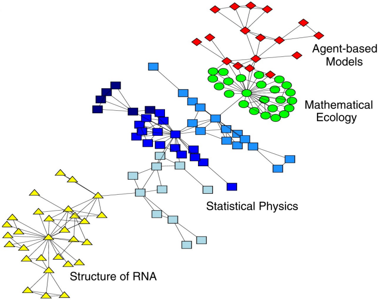

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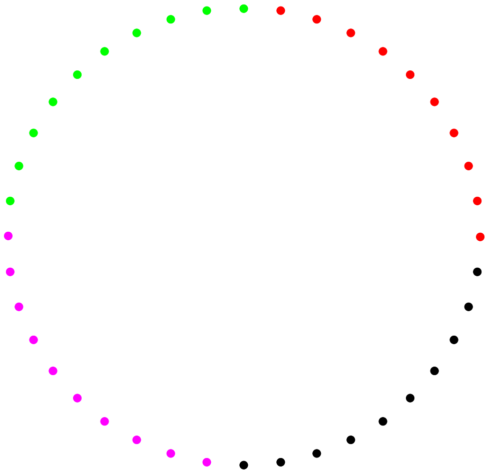
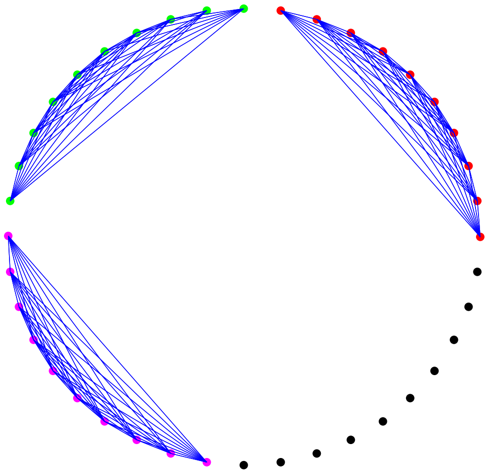
- Task: Find underlying communities based on the network topology
- Applications: Friend or movie recommendation in online social networks

Stochastic block model (Planted partition model)



$n = 40, K = 10, r = 3$

Stochastic block model (Planted partition model)



$p = 0.9$

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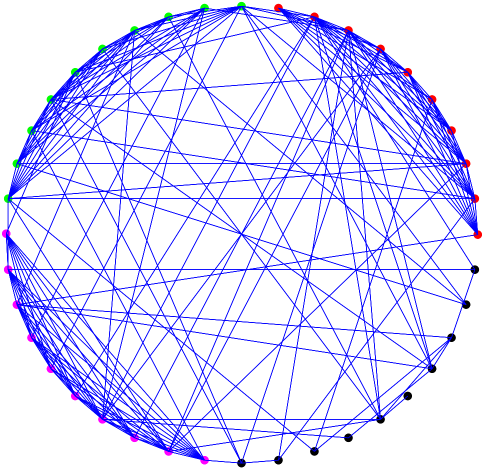

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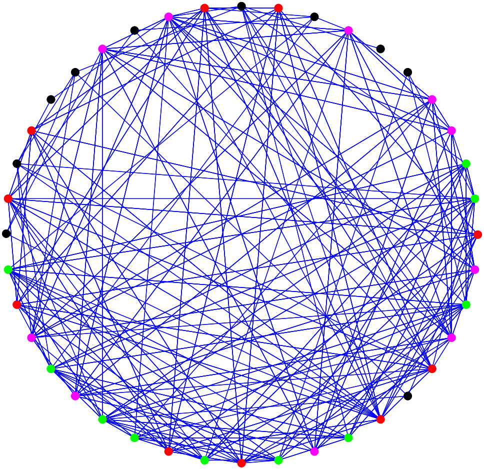

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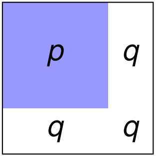
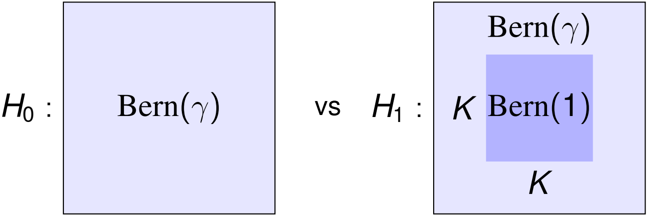
This paper focuses on $r = 1$: a single community



- One cluster of size K plus $n - K$ outliers
- Connectivity p within cluster and q otherwise
- Also known as *Planted Dense Subgraph* model
- $p = 1$, $q = \gamma$ corresponds to *Planted Clique* model

Planted clique hardness hypothesis



[Alon et al. '98] [Dekel et al. '10] [Deshpande-Montanari '13]...
Intermediate regime: $\log n \ll K \ll \sqrt{n}$, $\gamma = \Theta(1)$

- detection is possible but believed to have high computational complexity: [Alon et al. '11] [Feldman et al. '13]...

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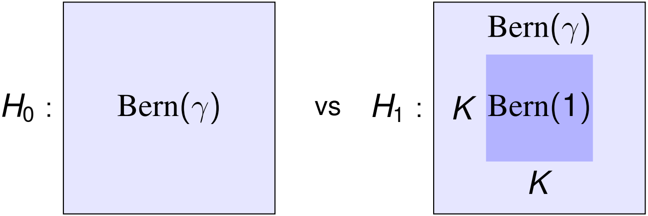

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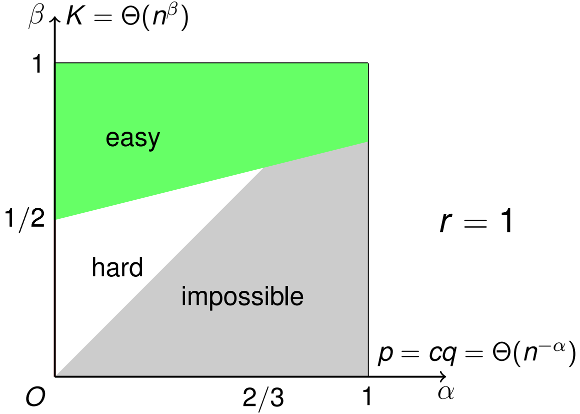
Intermediate regime: $\log n \ll K \ll \sqrt{n}$, $\gamma = \Theta(1)$

- detection is possible but believed to have high computational complexity: [Alon et al. '11] [Feldman et al. '13]...
- many (worst-case) hardness results assuming Planted Clique hardness with $\gamma = \frac{1}{2}$
  - detecting sparse principal component [Berthet-Rigollet '13]
  - detecting sparse submatrix [Ma-Wu '13]
  - cryptography [Applebaum et al. '10]: $\gamma = 2^{-\log^{0.99} n}$

Main result: Hardness for *detecting* a single cluster

Assuming Planted Clique hardness for any constant $\gamma > 0$



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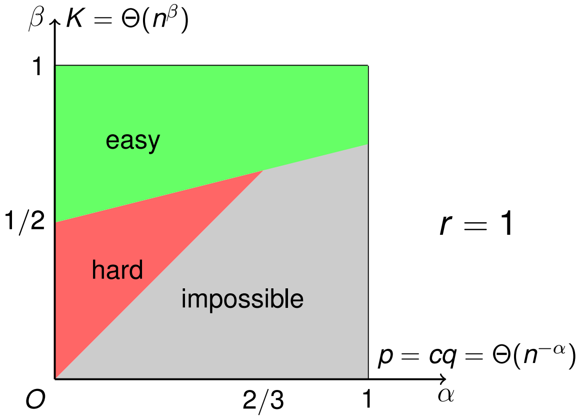
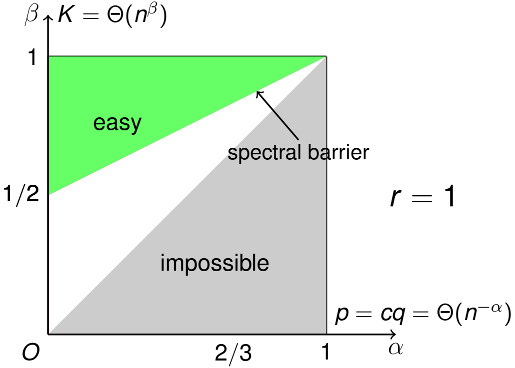


$r = 1$

Detecting a single cluster in the red regime is at least as hard as detecting a clique of size $K = o(\sqrt{n})$

Corollary: Hardness for *recovering* a single cluster

Can show: Hardness of detection implies hardness or recovery, so:
Assuming Planted Clique hardness for any constant $\gamma > 0$



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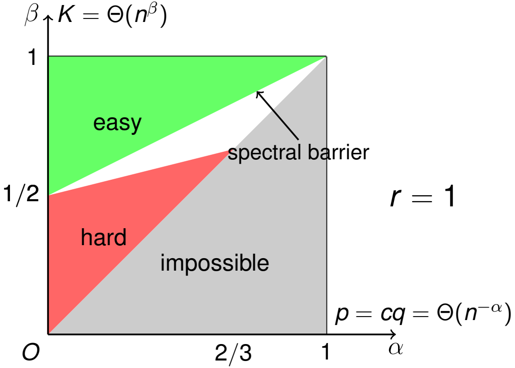
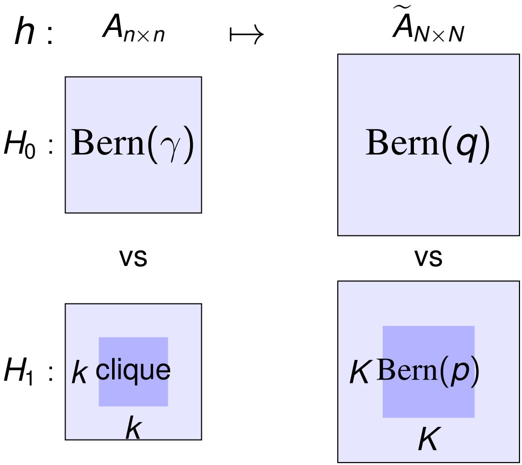


$r = 1$

Recovering a single cluster in the red regime is at least as hard as detecting a clique of size $K = o(\sqrt{n})$

Proof requires a polynomial time reduction



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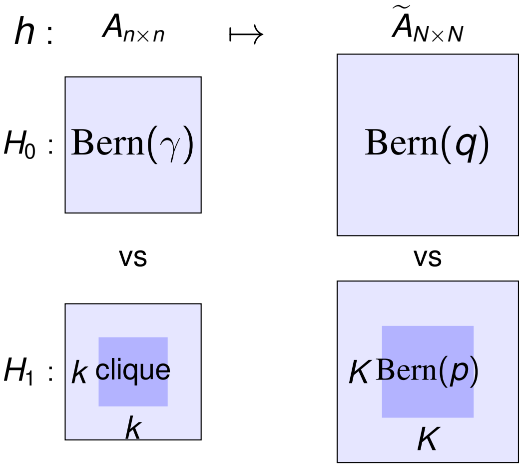
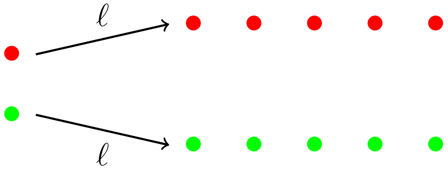


$h : A \mapsto \widetilde{A}$ is agnostic to the clique and can be computed in P-time

Given an integer ℓ, two probability distributions P, Q on $\{0, 1, \ldots, \ell^2\}$

Split each node
into ℓ new nodes
$N = n\ell, K = k\ell$



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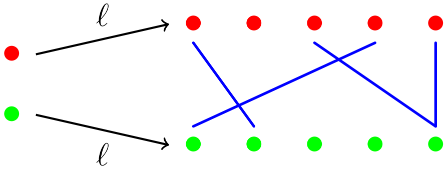

Assign edges with distributions P, Q

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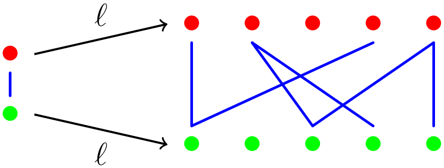

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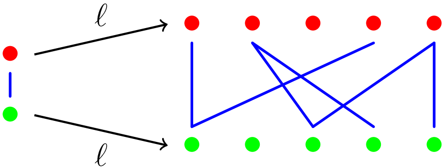

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$$H_0: \quad \mathrm{Bern}(\gamma) \qquad (1-\gamma)Q + \gamma P$$

$$H_1: \quad \mathrm{Bern}(1) \text{ (in-clique)} \qquad P \text{ (in-cluster)}$$

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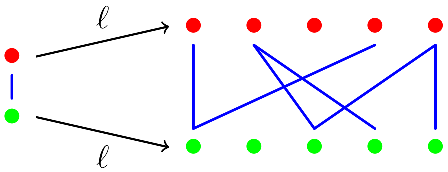

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How to choose P, Q?

Matching H_0: $(1-\gamma)Q + \gamma P = \mathrm{Binom}(\ell^2, q)$
Matching H_1 approximately: $P \approx \mathrm{Binom}(\ell^2, p)$ in total variation distance

Please see paper for more information and references

Thanks!